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A THEORETICAL STUDY
OF
PLANING CRAFT STABILITY

by

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B.Sc. University of New Brunswick
(1960)

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SUBMITTED IN PARTIAL FULFILLMENT
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A THEORETICAL STUDY OF PLANING CRAFT STABILITY

James R. McFarlane and Raymond N. Stoetzer

Submitted to the Department of Naval Architecture and Marine Engineering on 20 May, 1965 in partial fulfillment of the requirements for the degree of Naval Engineer and the degree of Master of Science in Naval Architecture and Marine Engineering.

ABSTRACT

Dynamic instability of planing craft on calm water, porpoising, is a phenomenon which has not been properly understood. Empirical relations are available for predicting the regime of stability. The relations, when compared, lead to conflicting design requirements to increase stability.

It is therefore desirable to develope a theoretical approach to the problem so that the effects of beam, deadrise angle, etc. on stability can be studied.

The results of the investigation imply that a decrease in deadrise angle, a decrease in beam and an increase in distance from LCG to transom result in an increase in stability. Changes in shaft angle and vertical height of the center of gravity and moment of inertia have very little effect on the stability of a boat while it is planing. However further investigation is required to verify these results.

In conjunction with this paper, a computer program was written which can be used in the design of planing craft to predict boat attitude, wetted surface area, drag and effective horse power. This program will be available for use in the XIII Department library.

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Thesis Supervisor: Philip Mandel

Title: Professor of Naval Architecture

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NOMENCLATURE

Standard Symbol	Definition	Program Symbol
b	Beam	BEAM
C_{lb}	Lift coefficient for prismatic surface	CLB
C_{lo}	Lift coefficient for zero deadrise surface	CLO
D_f	Drag force (lbs)	DRAG
F	Froude number $U_o / \sqrt{g \nabla \frac{1}{3}}$	
KG	Height of center of gravity above base line (ft)	VCG
l	Non-dimensionalizing length (ft)	BEAM
l_{cp}	Location of center of pressure forward of transom (ft)	CPL
l_{CG}	Location of center of gravity forward of transom (ft)	CG
lm	Mean wetted length (ft)	WETL
L_c	Length of wetted chine (ft)	WCHINE
L_k	Length of wetted keel (ft)	WKEEL
mr^2	Boat pitching moment of inertia about CG	YI
mX_G	Added inertia effect about Y-axis	VERYI
N	Normal force (lbs)	
T	Thrust	
	Mean velocity of flow past bottom	VM
	Angle of keel above horizontal (deg)	TRIM
	Non-dimensional velocity (fps)	U
	Wetted surface (ft^2)	S
β	Average deadrise angle (degrees)	BETA
Δ	Displacement (lbs)	W
ϵ	Shaft angle (degrees)	EPSIL
θ	Pitch angle (degrees)	TAU
λ	Ratio of mean wetted length to beam Note: Reference (10) uses this definition while reference (11) uses its reciprocal.	ASP

Standard Symbol	Definition	Program Symbol
ρ	Mass density	RHO
τ	Trim angle	TAU
$(m - Z_{w})$	Vertical force per unit vertical acceleration	A1
Z_{w}	Vertical force per unit vertical velocity	B1
Z_{z}	Vertical force per unit vertical displacement	C1
$(Z_{\dot{q}} + mX_{G})$	Vertical force per unit angular acceleration	D1
$(Z_{q} + U_{0}Z_{\dot{w}})$	Vertical force per unit angular velocity	E1
$(Z_{\theta} + U_{0}Z_{w})$	Vertical force per unit angular displacement	G1
$(I_{y} - M_{\dot{q}})$	Pitching moment per unit angular acceleration	A2
$(M_{q} + U_{0}M_{\dot{w}})$	Pitching moment per unit angular velocity	B2
$(M_{\theta} + U_{0}M_{w})$	Pitching moment per unit angular displacement	C2
$(M_{\dot{w}} + mX_{G})$	Pitching moment per unit vertical acceleration	D2
M_{w}	Pitching moment per unit vertical velocity	E2
M_{z}	Pitching moment per unit vertical displacement	G2
	$A1/(0.5 \cdot RHO \cdot l^{3})$	A11
	$B1/(0.5 \cdot RHO \cdot U \cdot l^{2})$	B11
	$C1/(0.5 \cdot RHO \cdot U^{2} \cdot l)$	C11
	$D1/(0.5 \cdot RHO \cdot l^{4})$	D11
	$E1/(0.5 \cdot RHO \cdot U \cdot l^{3})$	E11
	$G1/(0.5 \cdot RHO \cdot U^{2} \cdot l^{2})$	G11
	$A2/(0.5 \cdot RHO \cdot l^{5})$	A22
	$B2/(0.5 \cdot RHO \cdot U \cdot l^{4})$	B22
	$C2/(0.5 \cdot RHO \cdot U^{2} \cdot l^{3})$	C22
	$D2/(0.5 \cdot RHO \cdot l^{4})$	D22
	$E2/(0.5 \cdot RHO \cdot U \cdot l^{3})$	E22
	$G2/(0.5 \cdot RHO \cdot U^{2} \cdot l^{2})$	G22

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I. INTRODUCTION

The unstable motions of planing craft have been under study for many years and have been the subject of much literature (see bibliography). The ability to be able to predict the stability characteristics of a particular hull in the early stages of design is of importance to naval architects. A knowledge of the effects of variables such as beam, deadrise angle, etc. on stability would permit intelligent corrective action to be taken to increase the dynamic stability of existing craft.

The problem of planing craft stability involves many variables and empirical relations between some of the design variables have been developed to predict dynamic stability.

Two formulae recently developed emperically from experimental data, (2) and (12), result in conflicting design requirements to increase stability (see Appendix C). It is therefore desireable to develope a theoretical approach to the problem so that the effects of design variables can be determined independently of experimental data.

Perring (10) attempted a theoretical approach. His lack of success can be attributed to a number of causes. The foremost of these being lack of sufficient experimental and theoretical information to predict the stability derivatives accurately and the ommission of important terms.

II. THEORY

A planing hull, as a rigid body, has six degrees of freedom. This study treats the boat as a two degree of freedom system by investigating what is considered to be the most important motions, heave in the Z-direction and pitch about the Y-axis.

The equations of motion are nonlinear. To facilitate the solution of these equations it is necessary to linearize them.

Linearized equations of motion for ships have been developed by Abkowitz (1), Korvin-Korvosky (7) and others. Those of Abkowitz are most complete. If the coefficients, i.e. stability derivatives, are substituted into these equations and then the result transformed into the frequency domain, it should be possible to evaluate the stability by using the Routh criterion. (5).

The method used to predict stability or lack thereof proceeds as follows:

1. The stability derivatives are determined, see Appendix A.
2. The stability derivatives are substituted into the linearized equations for ship motion. (1)
3. The resulting equations are transformed by substitutions of the form $z = Zmax\ e^{st}$ and $\theta = \theta max\ e^{st}$.
4. An equation in S is obtained.
5. The Routh discriminant is evaluated for the fourth order equation in S, see Appendix A.

III. DESCRIPTION OF WORK ACCOMPLISHED

The firststep toward a solution was to determine the stability derivatives and combine them to form the coefficients of the linearized equations of motion, see Appendix A. The coefficients were then non-dimensionalized using beam as the non-dimensionalizing length (12).

A computer program was written to solve for the Routh discriminants, see Appendix H, using as input data the results from a series of tests run at the David Taylor Model Basin (2), see Table 3. The resulting discriminants were then plotted against speed showing a consistency in the directions of the paths, however there was no obvious difference between stable and unstable[1] boats, see Figure 9.

At this point, an attempt was made to determine the roots of the fourth order stability equation to examine their loci using a computer program from the MIT "SHARE" library. (SHARE No. 1514 RTSCH). RTSCH proved to be unsatisfactory. The answers obtained from this program are seriously in error even for the simplest of input equations.

It was then decided to vary in turn what seemed to be the most important variables: DIFB1, DIFC1, B2, D1, D2, E1, G1, and G2. This was done to determine the effect of changes in their magnitude on the Routh discriminant. From Figure 10 it can be seen that varying G2 roughly grouped the stable and unstable boats with the unstable group centered about G2 equal to 0.38 x G2 at the point of zero Routh discriminant. The program was then run with G2 equal to 0.38 x G2 so that the loci of the discriminants could be examined. The results are shown in Figure 11. Based on these results it was decided to in-

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[1] Unstable boats, as refered to in this paper, are those which porpoised at a F_∇ less than 6.0. Stable boats are those which had not porpoised before maximum test speed (2) was attained (F_∇ = 6.0).

vestigate G2 further. Since the term G2 is made up of two parts, force x $\frac{\partial(\text{arm})}{\partial z}$ + arm x $\frac{\partial(\text{force})}{\partial z}$, it was decided to vary these two parts independently to see if better correlation could be achieved in either of the two groups.[1] Correlation was not improved, see Figures 12 through 17. However it was observed that the discriminants for the stable boats changed very little with changes in G2, or its parts. On the other hand the discriminants of the unstable boats changed a great deal with changes in G2. This lead to the conclusion that there must be a term in the coefficients of the characteristic equation which overpowered G2 when the boat was stable but was of the same magnitude as G2 when the boat was unstable.

Based on information obtained thus far, it was decided to investigate each term of the Routh discriminant to see which of the coefficients were controlling for the stable and unstable boats. Values of the coefficients obtained from program 1 in Appendix H were inserted into each term manually and inspection of the results was unfruitful. No obvious difference could be detected between stable and unstable boats. It was concluded that the interaction was much more subtle.

A closer inspection of each term indicated that the coefficients $Z_{\dot{q}}$ and $M_{\dot{w}}$, D1 and D2, may interact with important effects and this became the final step in the investigation of the coefficients. The results are shown in Figures 18 through 20. At this point the investigation of the coefficients of the equations of motion was terminated because of time

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[1] At this point it was necessary to reduce the number of plots to three stable and three unstable boats in order both to simplify the plots and to make more efficient use of computer time. The unstable boats were selected by choosing two which had axis intercepts fairly close together and third whose intercept was remote from these (models 4665-3, 4666-13, and 4668-9 in Figure 10).

limitations.

Concurrently with the above work, a program was written to solve for all the hydrodynamic performance characteristics of the planing hull: planing angle, wetted surface, resistance, power requirements, and stability. This program uses design parameters as inputs and provides an easily readable output. Development of the criterion for the equilibrium planing condition is shown in Appendix E and details of the program are contained in Appendix H. In order to facilitate the writing of this program, it was necessary to determine an expression for mean bottom velocity based on an emperical function of deadrise angle, Appendix D.

The coefficients of the equations of motion and the Routh discriminant based on the computed planing conditions were compared with those based on experimental data. The result of this comparison is shown in Table 2.

The program was then run, for model 4668-9 with G2 equal to 0.38 x G2, varying BETAI, EPSILI, VCG, BEAM, CG, and YI in turn. The results were then plotted, Figure 21, so that a comparison of the relative effects of the variables could be made.

IV. DISCUSSION OF RESULTS

The plots of the discriminant versus speed obtained from the first calculations, Figure 9, are disappointing. According to these results most models were stable throughout the entire range of speeds investigated, contrary to the experimental results. The lack of agreement could be caused by one, or both, of the following:

(a) incorrect formulation of one or more of the stability derivatives
(b) neglecting the cross coupling effects of longitudinal motion.

Perring (10) indicated that inclusion of the longitudinal motion cross coupling effects had negligible effect on the outcome of his solution to the problem. It is possible that in the present, more refined solution, the magnitude of this cross coupling may become relevant.

The investigation of the effects of varying the magnitudes of the stability derivatives indicates that a solution to the problem may lie in this area. Although variations in Z_w, Z_z, and $(Z_\theta + u_0 \; Z_w)$, DIFB1, DIFC1, and G1, failed to yield any evidence of consistent influence, Figure 10 shows that variation of G2 produced a fairly consistent difference between stable and unstable boats. It is true that there is considerable scatter of the axis intercepts within each group, but there is an undeniable consistency in the grouping. It is also of interest to note there is much less scatter in the stable group than in the unstable group. The grouping of the stable boats cannot be attributed to their equal Froude number. An examination of the unstable boat grouping, Figure 10, indicates that models 4665-3 and 4668-9 intercept the axis at the same point with F_∇ of 3.24 and 5.03 respectively whereas model 4666-17, which has a F_∇ of 5.01 (essentially equal to that of model 4668-9), has an intersection remote from the preceeding two.

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The investigation of the loci of the discriminants with C2 equal to 0.55 x C2, Figure 11, show that the original curves, shown in Figure 9, now bend down toward negative values of discriminants as speed increases. Some of the unstable models have loci consistent with experimental data, i.e. the Routh discriminant heads toward negative values as the porpoising speed is approached however, the boats which were stable throughout their test range have loci lying completely below the axis (negative discriminants indicate instability). This indicates that simply multiplying G2 by a single factor does not produce reliable Routh discriminants.

The investigation of the effects of varying the two components of G2 separately, Figures 12 through 17, although not producing better correlation within the stable or unstable groups does point out the small effect that these variations have on the intercepts of the stable group compared to their effect on the unstable boats. This information, as it stands, indicates that other terms in the equations of motion are more powerful at stable speeds but that, at the porpoising speed, G2 is a powerful term.

The results of the investigation of the simultaneous variations of D1 and D2, see Figures 18 through 20, point the way to what may be a valuable area for further study. The first useful bit of information obtained is the fact that $Z_{\dot{q}}$, D1, has very little effect on the magnitude of the Routh discriminant and that $M_{\dot{w}}$, D2, has a large effect. The most important result of this investigation is the fact that the axis intercepts of models 4666-13 and 4668-9 have been reversed in their relative positions from what they were when the coefficient G2 was varied. This means that a simultaneous variation of D2 and G2 may cause these two extreme boats to cross the axis at the same point and thereby correlate the unstable group.

Correlating the results in this manner does not really solve the problem. The stability indicator, Routh discriminant, is not reliable, as the discussion of Figure 11, Appendix F, has shown. Further work is not bend down toward negative values of discriminants as speed increases. Some of the unstable models were most consistent with experimental data, i.e., the Routh discriminant heads toward negative values as the porpoising speed is approached however, the boats which were stable throughout their test range have been lying completely below the axis (negative discriminants indicate instability). This indicates that simply multiplying C2 by a single factor does not produce reliable Routh discriminants.

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Correlating the results in this manner does not really solve the problem. The stability indicator, Routh discriminant, is not reliable, as the discussion of Figure 11, Appendix F has shown. Further work is required to produce better agreement between the intercepts of the discriminants and experimental data. An experimental investigation of the individual stability derivatives for comparison with the theoretically developed derivatives would be helpful in locating the terms of the equations of motion which need to be reevaluated.

The program, which solves for the hydrodynamic performance characteristics of planing boats, yields information of importance to design.

Starting with attitude, wetted keel length, wetted chine length, and drag, it can be seen, Table 1, that there is good agreement between theory and experiment for boat attitude and drag. The theoretical values of wetted keel length and wetted chine length are larger than the experimental values.

A plot of $\frac{\text{WKEEL - WCHINE}}{\text{b}}$ vs TRIM comparing theory (12) and values calculated from the experimental results of (2), Figure 22, indicates that the mean line of data points lies above the theoretical line for this group of boats, Table 3. The largest errors occur at the largest angles of attack.

The expression developed for mean bottom velocity, Appendix D, yields results which compare very accurately with graphs shown in Figure 7.

The comparison of derivatives calculated directly from the program shows good agreement with the exception of C2 and E2, see Table 2. This error was most likely caused by the difference in actual and calculated wetted length.

The results of the variation of BETAI, EPSILI, BEAM, YI and CG, shown on Figure 21, can not be conclusive because of inconsistencies which have been found in the discriminant. However Figure 21 does show that the beam, deadrise angle and longitudinal position of the center of

gravity have the largest effect on stability. The implication of Figure 21 that a decrease in beam increases stability agrees with (12). The inference that an increase in deadrise angle results in a decrease in stability is, at first glance, distressing. However an inspection of Figure 16 of reference (12) indicates that this may well be the case for the boat examined. An increase in deadrise angle results in an increase in trim and a decrease in the lift coefficient. Both of the latter effects are destabilizing. If the destabilizing influence caused by the change of trim and of the lift coefficient is greater than the stabilizing effect of the increase in deadrise angle, the boat will be destabilized.

The results indicate that moving the center of gravity forward increases the range of stability. This forward movement results in a decrease in trim and it is known that a decrease in trim results in an increase in the range of stability.

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V. CONCLUSIONS

Unfortunately this thesis does not go far enough to solve the problem of predicting the stability characteristics of a planing craft in the design stage. The relative magnitudes of the force and moment derivatives which make up the coefficients of the equations of motion give rise to inconsistencies in predicting stability which have not been resolved.

This study does indicate however that variations in the following quantities have a minor effect on the magnitude of the stability indicator (Routh discriminant):

(a) change in lift coefficient x area with respect to vertical velocity (DIFB1)

(b) change in lift coefficient x area with respect to vertical position (DIFC1)

(c) change in vertical force with respect to the angular acceleration about the pitch axis ($Z_{\dot{q}}$ or D1)

(d) change in vertical force with respect to the angular velocity about the pitch axis ($Z_q + u_0 .$ VERM or E1)

(e) change in vertical force with respect to trim ($Z_\theta + u_0 \cdot Z_w$ or G1)

(f) change in pitching moment with respect to velocity in the heave direction ($M_q + u_0 \cdot M_{\dot{w}}$ or B2)

and that the following have a major effect:

(a) change in pitching moment with respect to vertical acceleration ($M_{\dot{w}}$ or D2)

(b) change in pitching moment with respect to vertical position (M_z or G2).

The computer program, developed as part of this thesis, which solves for the other hydrodynamic performance characteristics of planing craft is able to reproduce experimental results with minor limitations. The expressions used in computing wetted length of chine and keel do not

yield accurate results. This causes an error in wetted length and aspect ratio which is further reflected in some of the coefficients of the motion equations.

The expression developed for computing the mean bottom velocity yields consistently good results.

The results of the study of the variations of design parameters generated by means of computer Program 2, Figure 21, show that the range of stability may be increased by decreasing deadrise angle, decreasing beam and moving the center of gravity forward. Changes in the vertical center of gravity, moment of inertia and shaft angle have minor effects on the range of stability.

In spite of the inconclusive results of the stability investigation, there is substantial indication that the stability problem can be solved. The solution of this problem will utilize results like those shown in Figure 21 in conjunction with Program 2. This should provide an extremely useful design tool for optimizing planing craft design. For example; assume that it is desired to design a planing hull to operate at 40 knots. The design procedure would proceed along the following lines:

(a) Run computer Program 2 for a number of combinations of beam, deadrise angle, and the longitudinal position of the center of gravity obtaining drag information for all combinations which yield designs stable to 40 knots.

(b) From the data thus obtained, develop a family of curves for each deadrise angle by plotting drag versus beam for several locations of center of gravity.

(c) Choose the design for minimum drag for a 40 knot planing hull from the curves.

VI. RECOMMENDATIONS

1. Repeat the study described herein using three degrees of freedom: pitch, heave, and surge.

2. Develop a more accurate method of predicting wetted length of keel and wetted length of chine for a planing surface with deadrise angle based on experimental data.

3. Make an experimental investigation of the force and moment derivatives for comparison with those developed theoretically.

4. An examination of the loci of the roots of the characteristic equation in the S-plane would produce valuable results once the inconsistencies in predicting stability are ironed out. An investigation of this sort would show the effect that variations in design parameters have on how the roots approach and cross the imaginary axis. This requires a more accurate root extraction computer program than was available to the authors.

APPENDIX A

Equations representing force and moment equilibrium for a ship can be expressed in the form: (1)

$$A1Z + B1Z + C1Z + D1\theta + E1\theta + G1\theta = Fe^{i\omega t} \text{ .... forces}$$

$$A2\theta + B2\theta + C2\theta + D2Z + E2Z + G2Z = Me^{i\omega t} \text{ .... moments}$$

For the case under consideration here, smooth water, there are no external excitations. Therefore $Fe^{i\omega t}$ and $Me^{i\omega t}$ are both equal to zero.

By substituting the transforms Ze^{st} and θe^{st} into the force and moment equations we can obtain a characteristic equation of the form:

$$AA.S^4 + BB.S^3 + CC.S^2 + DD.S + EE = 0$$

Where:

$$AA = 1.$$

$$BB = \frac{A22.B11 + A11.B22 - D22.E11 - E22.D11}{A11.A22 - D11.D22}$$

$$CC = \frac{A22.C11 + B22.B11 + A11.C22 - D22.G11 - E22.E11 - G22.D11}{A11\ \ A22 - D11\ \ D22}$$

$$DD = \frac{B22.C11 + B11.C22 - E22.G11 - G22.E11}{A11.A22 - D11.D22}$$

$$EE = \frac{C22.C11 - G22.G11}{A11.A22 - D11.D22}$$

If the Routh criterion is applied to the characteristic equation it should be possible to evaluate the stability of the boat (5). The criterion indicates that a boat will be stable and nonoscillatory in the steady state if:

$$BB.CC.DD - AA.DD^2 - BB^2.EE > 0$$

Negative Routh discriminants are not meaningful for the case under study. Negative roots denote instability, instability implies motion, and motion in this case implies nonlinearity. Since the method is based on linearized equations, the last meaningful Routh discriminant is zero.

THE DERIVATION OF COEFFICIENTS FOR FORCE AND MOMENT EQUATIONS:

A1

A1 and A2 will have to have terms containing added mass and added inertia respectively.* This is necessary because the added mass and added inertia may be of the same order of magnitude as that of the boat itself.

A1 = W + VERM

B1

$B1 = Z_w$

$Z_w = \frac{\partial Z}{\partial w} = \frac{\partial \theta}{\partial w} . \frac{\partial Z}{\partial \theta}$

$\frac{\partial \theta}{\partial w} = \frac{1}{V}$

$\frac{\partial Z}{\partial \theta} = \frac{1}{2} \text{RHO}. u_o^2 . \frac{\partial CLA}{\partial \theta}$

$Z_w = 0.5 \ \text{RHO} \ u_o \ \text{DIFB1}$

$B1 = Z_w$

where:

$CLA = C_1 \ A$ (lift coefficient x area)

$DIFB1 = \frac{\partial CLA}{\partial \theta}$ (DIFB1 is the symbol used in the computer program)

C1

$C1 = Z_z$

$Z_z = \frac{\partial Z}{\partial z} = \frac{\partial \text{(lift)}}{\partial z} = \frac{1}{2} \text{RHO}. u_o^2 . \frac{\partial (CLA)}{\partial z}$

$Z_z = 0.5 \ \text{RHO} \ u_o^2 \ \text{DIFC1}$

$C1 = Z_z$

---

* See Appendix B.

where:

DIFC1 = computer program symbol for $\frac{\partial CLA}{\partial z}$

<u>D1</u>

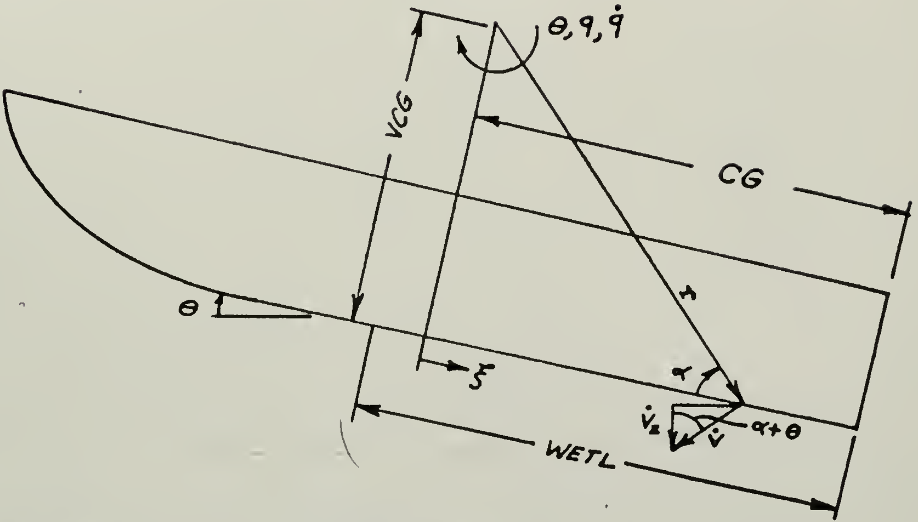


FIGURE 1

$D1 = Z_{\dot{q}}$

$Z_{\dot{q}} = \frac{\partial Z}{\partial \dot{q}}$

$r = \frac{VCG}{\sin a}$

Limits on a :

$\cot^{-1}\left(\frac{CG - WETL}{VCG}\right) \rightarrow \frac{\pi}{2} \rightarrow \cot^{-1}\left(\frac{CG}{VCG}\right)$

$\dot{v} = r.\ \dot{q} = \frac{VCG.\ \dot{q}}{\sin a}$

$\dot{v}_z = \frac{VCG.\ \dot{q}.\ \cos(\theta + a)}{\sin a}$

$d\ Z = d\ (m.\ \dot{w}) = \frac{\pi}{2}\ .\ RHO.\ BEAM^2.\ d\ \xi\ \left(\frac{VCG}{\sin a}.\ \cos(\theta + a)\right)\ \dot{q}^{*}$

$\xi = VCG\ .\ \cot a$

---

[*] See reference (9), page 420, Fig. 62A.

$$d\,\xi = -VCG.\ \csc^2\alpha\, d\alpha$$

$$\frac{Z}{\dot{q}} = -\frac{\pi}{2}.\ RHO\ .\ BEAM^2.\ VCG^2 \int_{\xi = CG - WETL}^{\xi = CG} \frac{\cos(\theta + \alpha)\, d\alpha}{\sin^3 \alpha}$$

$$Z_{\dot{q}} = \frac{Z}{\dot{q}}$$

$$Z_{\dot{q}} = \frac{\pi}{2}.\ RHO\ .\ BEAM^2.\ VCG\ .\ WETL.\left[\frac{\cos\theta}{VCG}(CG - \frac{WETL}{2}) - \sin\theta\right]$$

$$D1 = Z_{\dot{q}}$$

<u>E1</u>

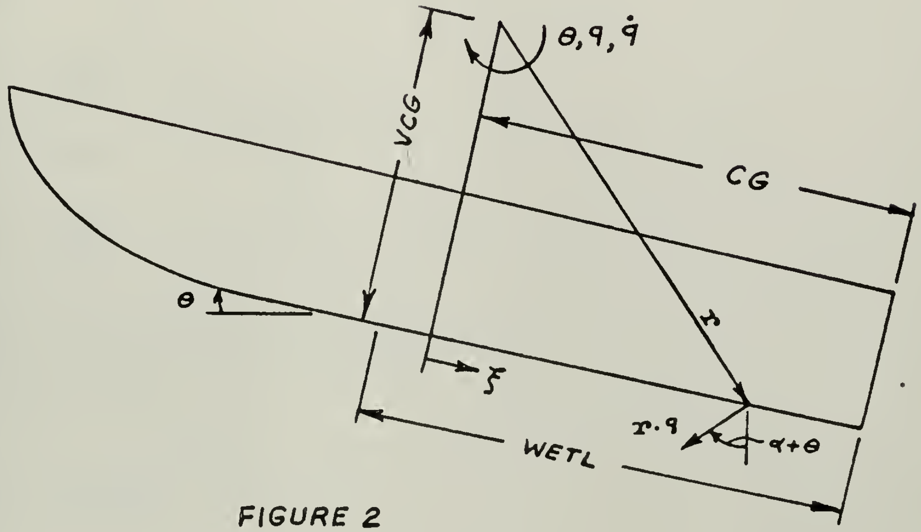


FIGURE 2

$$E1 = Z_q + u_0.\ Z_{\dot{w}}$$

$$Z_q = \frac{\partial Z}{\partial q} = \frac{\partial w}{\partial q}.\ \frac{\partial \theta}{\partial w}.\ \frac{\partial Z}{\partial \theta} = \frac{\partial w}{\partial q}.\ B1$$

We now wish to express q as an effective velocity, w.

$$r = \frac{VCG}{\sin\alpha}$$

$$v = r\ .\ q\ .\ \cos(\alpha + \theta) = VCG\ .\ q\,(\cos\theta\ .\ \cot\alpha - \sin\theta)$$

d (v. a) = increment of velocity x area

$$= VCG\ .\ q\ .\ (\cos\theta\ .\ \cot\alpha\ - \sin\theta)\ d\,\xi\ .\ BEAM$$

$$\bar{V}.A = -VCG^2.\ BEAM\ .\ q.\left[\cos\theta\int\cot\alpha\ .\ \csc^2\alpha\, d\,\alpha\ - \sin\theta\int\csc^2\alpha\, d\alpha\right]\Bigg|_{\xi=CG-WETL}^{\xi=CG}$$

where: A = wetted area = WETL . BEAM

$$\bar{V}.A = -WETL\ .\ BEAM\ .\ q.\left[\cos\theta\ (0.5\ WETL - CG) + VCG\ .\ \sin\theta\right]$$

$$Z_q = B1\left[\cos\theta\ (CG - 0.5\ WETL) - VCG\ .\ \sin\theta\right]$$

$$E1 = Z_q + u_o\ .\ VERM$$

G1

$$G1 = Z_\theta + u_o .\ Z_w$$

$$Lift = \frac{1}{2} .\ RHO\ .\ u_o^2\ .\ CLA = Z$$

$$\frac{\partial Z}{\partial\theta} = \frac{1}{2}\ .\ RHO\ .\ u_o^2\ \frac{\partial(CLA)}{\partial\theta}$$

$$Z_\theta = \frac{1}{2}\ .\ RHO\ .\ u_o^2\ DIFB1 = u_o\ B1$$

$$G1 = Z_\theta + u_o\ Z_w = u_o\ B1 + u_o\ B1$$

$$= 2.\ u_o\ .\ B1$$

A2 = YI + VERYI*

where:

YI = pitching moment of inertia

VERYI = added inertia.

---

* See Appendix B for development of VERYI.

B2

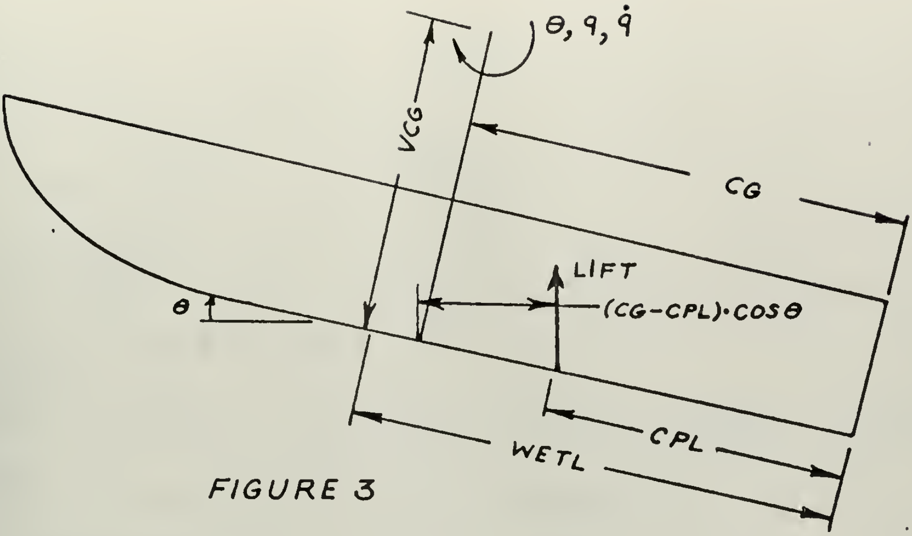


FIGURE 3

$$B2 = M_q + u_o \,.\, M_{\dot{w}}$$

$$M_q = \frac{\partial(\text{force} \,.\, \text{arm})}{\partial q} = \text{arm}\ \frac{\partial(\text{force})}{\partial q} + \text{force}\ \cancelto{o}{\frac{\partial\,(\text{arm})}{\partial q}}$$

$$= [\,(CG - CPL)\cos\theta - VCG \,.\, \sin\theta\,]\ \frac{\partial Z}{\partial q}$$

$$= [\,(CG - CPL)\cos\theta - VCG \,.\, \sin\theta\,].\ E1$$

$$B2 = M_q + u_o M_{\dot{w}} = M_q + u_o \,.\, D2$$

C2

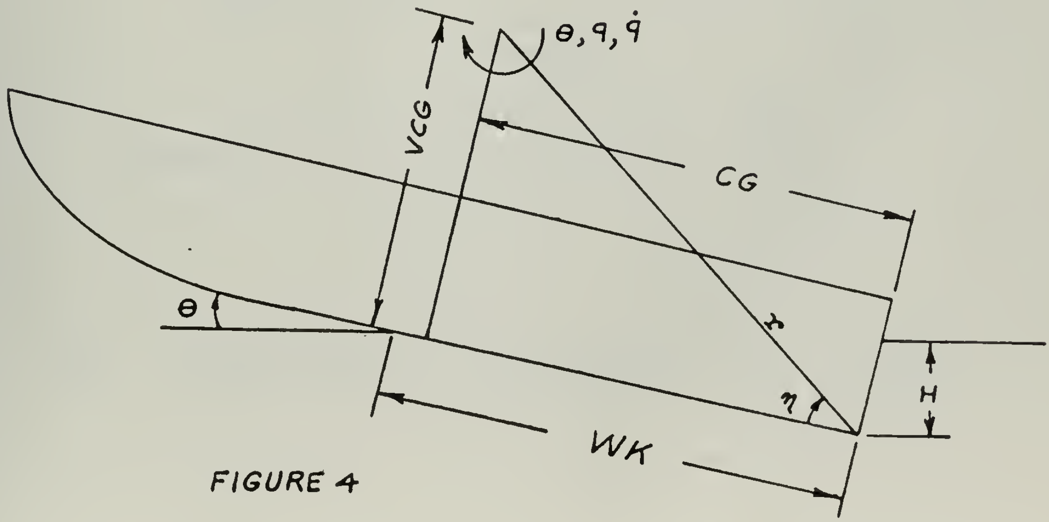


FIGURE 4

$$r = (VCG^2 - CG^2)^{\frac{1}{2}}$$

$$WK = H/\sin\theta \quad , \quad \eta = \sin^{-1}(VCG/r)$$

$$H = r\cdot\sin(\eta + \theta) - (VCG - H_{\theta=0})$$

$$M_\theta = \text{force}\ \frac{\partial(\text{arm})}{\partial\theta} + \text{arm}\ \frac{\partial(\text{force})}{\partial\theta}$$

$$= W\ \frac{\partial(\text{arm})}{\partial\theta} + [(CG - CPL)\cos\theta - VCG\cdot\sin\theta]\cdot G1$$

Assuming: $CPL = C_1 \cdot WK$

$$\frac{\partial(\text{arm})}{\partial\theta} = - CG\cdot\sin\theta - VCG\cdot\cos\theta - C_1\cdot\cos\theta\ \frac{\partial(WK)}{\partial\theta} + C_1\cdot WK\cdot\sin\theta$$

$$\frac{\partial(WK)}{\partial\theta} = - r\cdot\sin\eta\cdot\csc^2\theta + VCG\cdot\cot\theta\cdot\csc\theta$$

$$= VCG\cdot\csc\theta\,(\cot\theta - \csc\theta)$$

$$\frac{\partial(\text{arm})}{\partial\theta} = CPL\left[\sin\theta - \frac{VCG}{WK}\cdot\cot\theta\,(\cot\theta - \csc\theta)\right] - CG\cdot\sin\theta - VCG\cdot\cos\theta$$

$$C2 = M_\theta + u_o\ M_w = M_\theta + u_o\ E2$$

D2

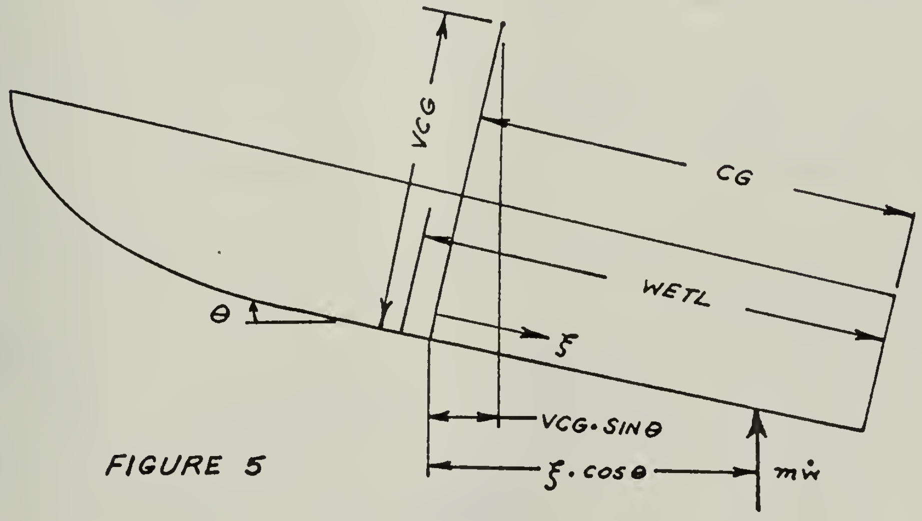


FIGURE 5

$$D2 = M_{\dot{w}}$$

$$M_{\dot{w}} = \frac{\partial M}{\partial \dot{w}} = \frac{\partial(\text{force}\cdot\text{arm})}{\partial \dot{w}} = \text{arm}\ \frac{\partial(\text{force})}{\partial\ \dot{w}} + \text{force}\ \overset{0}{\cancel{\frac{\partial(\text{arm})}{\partial\ \dot{w}}}}$$

$$dM = \frac{\pi}{2} \cdot RHO \cdot BEAM^2 \cdot d\xi \, (\xi \cdot \cos\theta - VCG \cdot \sin\theta) \cdot \dot{w}$$

Note: Incremental force = d (added mass $\cdot \dot{w}$)

added mass term taken from reference (11), page 420, Figure 62A, for an increment of length.

$$M_{\dot{w}} = \frac{\pi}{2} \cdot RHO \cdot BEAM^2 \int_{CG - WETL}^{CG} (\xi \cdot \cos\theta - VCG \cdot \sin\theta) \, d\xi$$

$$= \frac{\pi}{2} \cdot RHO \cdot BEAM^2 \cdot WETL \, [\cos\theta \, (CG - .5 \, WETL) - VCG \cdot \sin\theta \,]$$

$$D2 = M_{\dot{w}}$$

E2

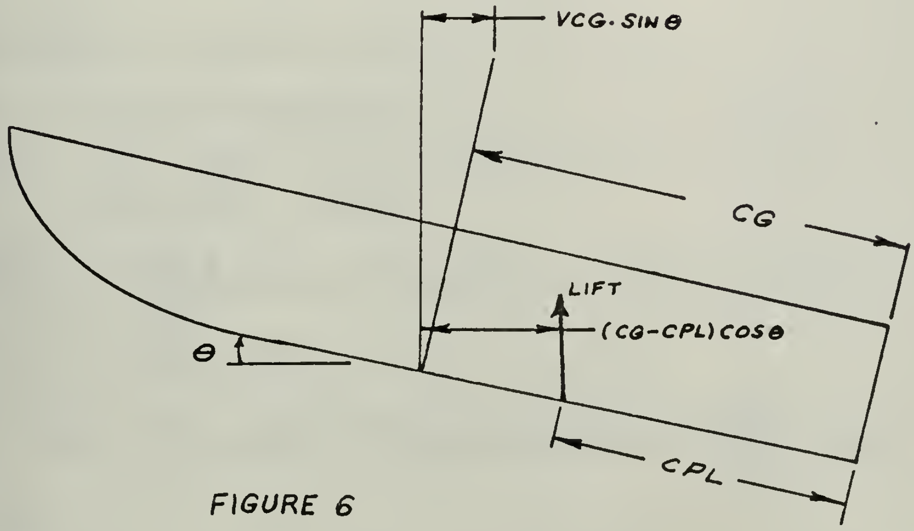


FIGURE 6

$$E2 = M_w$$

$$M_w = \frac{\partial M}{\partial w} = \frac{\partial \, (force \cdot arm)}{\partial w}$$

$$= arm \, \frac{\partial (force)}{\partial w} + force \, \cancelto{0}{\frac{\partial \, (arm)}{\partial w}}$$

$$= [(CG - CPL) \cos\theta \; - VCG \cdot \sin\theta \,] \, \frac{\partial \, (lift)}{\partial w}$$

$$M_w = [\,(CG - CPL) \cos\theta - VCG \cdot \sin\theta \,] \; B1$$

$$E2 = M_w$$

G2

See Figure 4.

$$G2 = M_z$$

$$M_z = \frac{\partial M}{\partial z} = \frac{\partial(\text{force}.\text{arm})}{\partial z}$$

$$= \text{force}\ \frac{\partial(\text{arm})}{\partial z} + \text{arm}\ \frac{\partial(\text{force})}{\partial z}$$

$$\frac{\partial(\text{arm})}{\partial z} = -\cos\theta\ \frac{\partial(\text{CPL})}{\partial z}$$

assuming: $\text{CPL} \approx C_1 \cdot \text{WK}$

where C_1 is an arbitrary coefficient.

$$\text{WK} = \frac{H}{\sin\theta}$$

where H = draft at transom.

$$\frac{\partial(\text{WK})}{\partial z} = \frac{1}{\sin\theta}\ \frac{\partial H}{\partial z} = \frac{1}{\sin\theta}$$

$$\frac{\partial(\text{arm})}{\partial z} = -\frac{\cos\theta}{\sin\theta}\ C_1 = -C_1 \cot\theta$$

$$= -\frac{\text{CPL}}{\text{WK}}\ \cot\theta$$

$$M_z = \left[(\text{CG} - \text{CPL})\cos\theta - \text{VCG}\sin\theta\right] C_1 - W\ \frac{\text{CPL}}{\text{WK}}\ \cot\theta$$

$$G2 = M_z.$$

Note: Computer program determines $\frac{\partial(\text{arm})}{\partial z}$ by incrementing variables in the equation $\text{CPL} = 0.75 - \cfrac{1}{\cfrac{C}{5.21(\frac{v}{\lambda})^2 + 2.39}}$

developed in reference (10), page 16.

APPENDIX B

Solving for Added Mass:

Because of a lack of information about deadrise surfaces the added mass is calculated as for a submerged elliptic cylinder. The result will be divided in half and the minor axes of the ellipse will be set to zero. This is assumed to be a suitable approximation of the added mass of a deadrise hull at the water surface.

For flow past an elliptic cylinder (page 251, (8):

$$w = \frac{A}{\zeta} + \frac{B}{\zeta^2}$$

where: w = complex potential

$\zeta = e^{i\eta}$, a unit circle

$A = U\,(b \cos \alpha + i\, a \sin \alpha)$

$B = \frac{i}{4}\,\omega\,(a^2 - b^2)$

For the case in question where only the vertical oscillation is being considered,

$\omega = 0$

so: $$w = \frac{A}{\zeta} = \frac{U\,(b \cos \alpha + i\, a \sin \alpha)}{\zeta}$$

$$\overline{w} = \overline{A}\,\zeta$$

$$\frac{d\overline{w}}{d\zeta} = \overline{A}$$

Now: $$T = -\frac{1}{4}\, i\rho \int_{(c)} \frac{A\overline{A}}{\zeta}\, d\zeta$$ T = kinetic energy

$$= -\frac{1}{4}\, i\rho\, [2\pi i \text{ (the residue } A\overline{A})]$$

$$= -\frac{1}{4}\, i\rho\, 2\pi i\, U^2 (b^2 \cos^2 \alpha + a^2 \sin^2 \alpha)$$

For this case $\alpha = 90°$, $b = 0$ and $T = \frac{U^2}{2} \rho \pi a^2$

but $T = \frac{1}{2} M U^2$

then 2 VERM $= \rho \pi a^2$ per unit width

and VERM $= \frac{\rho\pi}{2}$ (WETL)^2(BEAM)

The Added Inertia:

The problem here is to determine the added inertia for a body rotating about some point other than its center.

For rotation about a point other than the center of the body the complex potential is given by:

$$w - i \omega Z_0 \bar{Z}$$

In this case $Z_0 = \bar{Z}$ because it is on the real axis.

$$Z = a \cos \eta + i b \sin \eta$$

then

$$w = \frac{A}{\zeta} + \frac{B}{\zeta^2} - i \omega Z_0 (a \cos \eta + i b \sin \eta)$$

$$\bar{w} = \bar{A}\zeta + B\zeta^2 + i \omega Z_0 a \cos \eta + b \omega Z_0 \sin \eta$$

$$\frac{d\bar{w}}{d\zeta} = \bar{A} + 2\bar{B}\zeta - i \omega Z_0 a \sin \eta \frac{d\eta}{d\zeta} + b \omega Z_0 \cos \eta \frac{d\eta}{d\zeta}$$

$$\zeta = e^{i\eta}$$

$$\frac{d\zeta}{d\eta} = i e^{i\eta} = \frac{1}{i\zeta}$$

$$\frac{d\bar{w}}{d\zeta} = \bar{A} + 2\bar{B}\zeta + \frac{\omega Z_0}{\zeta} \left(\frac{1}{i} b \cos \eta - a \sin \eta\right)$$

$$wdw = \frac{A\bar{A}}{\zeta} + \frac{\bar{A}B}{\zeta^2} - i \bar{A} \omega Z_0 (a \cos \eta + i b \sin \eta)$$

$$+ 2 \bar{B}A + \frac{2\bar{B}B}{\zeta} - i\, 2 \bar{B} \zeta \omega Z_0 (a \cos \eta + i b \sin \eta)$$

$$- \frac{i\omega^2 Z_0^2}{\zeta} (a \cos \eta + i b \sin \eta) \left(\frac{1}{i} b \cos \eta - a \sin \eta\right)$$

$$T = - \frac{1}{4} i \rho \int w d\bar{w}$$

$$= - \frac{1}{4} i \rho \left[\int_{(c)} \frac{\bar{A}A}{\zeta} + 2 \frac{\bar{B}B}{\zeta} + 2 \bar{B}A + \frac{AB}{\zeta^2} d\zeta \right.$$

$$+ \int_{(c)} - \frac{i\omega Z_0^2}{\zeta} (a \cos \eta + i b \sin \eta) \left(\frac{1}{i} b \cos \eta - a \sin \eta\right) d\zeta$$

$$+ \int_{(c)} - i \bar{A} \omega Z_0 (a \cos \eta + i b \sin \eta) d\zeta$$

$$\left. + \int_{(c)} - i\, 2 \bar{E} \zeta \omega Z_0 (a \cos \eta + i b \sin \eta) d\zeta \right]$$

$$= F1 + F2 + F3 + F4$$

The Added Inertia:

The problem here is to determine the added inertia for a body rotating about some point other than its center.

For rotation about a point other than the centre of the body the complex potential is given by:

$$w - i \omega Z_0 \bar{Z}$$

In this case $Z_0 = \bar{Z}_0$ because it is on the real axis.

$$\bar{Z} = a \cos \eta + i b \sin \eta$$

then

$$w = \frac{A}{\zeta} + \frac{B}{\zeta^2} - i \omega Z_0 (a \cos \eta + i b \sin \eta)$$

$$\bar{w} = \bar{A}\zeta + \bar{B}\zeta^2 - i \omega Z_0 a \cos \eta + b \omega Z_0 \sin \eta$$

$$\frac{d\bar{w}}{d\zeta} = \bar{A} + 2\bar{B}\zeta - i \omega Z_0 a \sin \eta \frac{d\eta}{d\zeta} + b \omega Z_0 \cos \eta \frac{d\eta}{d\zeta}$$

$$\zeta = e^{i\eta}$$

$$\frac{d\zeta}{d\eta} = i e^{i\eta} = \frac{1}{i\zeta}$$

$$\frac{d\bar{w}}{d\zeta} = \bar{A} + 2\bar{B}\zeta + \frac{\omega Z_0}{\zeta}\left(\frac{1}{i} b \cos \eta - a \sin \eta\right)$$

$$w d\bar{w} = \frac{A\bar{A}}{\zeta} + \frac{\bar{A}B}{\zeta^2} - i \bar{A} \omega Z_0 (a \cos \eta + i b \sin \eta)$$

$$+ 2 \bar{B}A + \frac{2B\bar{B}}{\zeta} - i 2 \bar{B} \zeta \omega Z_0 (a \cos \eta + i b \sin \eta)$$

$$- \frac{i\omega^2 Z_0^2}{\zeta} (a \cos \eta + i b \sin \eta)\left(\frac{1}{i} b \cos \eta - a \sin \eta\right)$$

$$T = -\frac{1}{4} i \rho \int w d\bar{w}$$

$$= -\frac{1}{4} i \rho \Bigg[\int_{(c)} \frac{A\bar{A}}{\zeta} + 2 \frac{B\bar{B}}{\zeta} + 2 \bar{B}A + \frac{AB}{\zeta^2} \, d\zeta$$

$$+ \int_{(c)} - \frac{i\omega Z_0^2}{\zeta} (a \cos \eta + i b \sin \eta)\left(\frac{1}{i} b \cos \eta - a \sin \eta\right) d\zeta$$

$$+ \int_{(c)} - i \bar{A} \omega Z_0 (a \cos \eta + i b \sin \eta) \, d\zeta$$

$$+ \int_{(c)} - i 2 \bar{B} \zeta \omega Z_0 (a \cos \eta + i b \sin \eta) \, d\zeta \Bigg]$$

$$= T_1 + T_2 + T_3 + T_4$$

$F1 = 2\pi i (A\bar{A} + 2 B\bar{B})$

$F2 = 2\pi i \omega^2 Z_0^2 a b + \int \frac{i (b^2 - a^2)}{\zeta} \frac{\sin 2\eta}{2} d\zeta$

but $\zeta = e\, i\eta$

$d\zeta = i e^{i\eta}$ limits: $0 \rightarrow 2\pi$

$$F2 = 2\pi i \omega^2 Z_0^2 a b + \int_0^{2\pi} i \frac{(b^2 - a^2)}{e^{i\eta}} \cancelto{0}{\frac{\sin 2\eta}{2}} e^{i\eta} d\eta$$

$F2 = 2\pi i \omega^2 Z_0^2 a b.$

$$F3 = i \bar{A} \omega Z_0 \int_0^{2\pi} (a \cos \eta + i b \sin \eta)(\cos \eta + i \sin \eta) d\eta$$

$$= i \pi \bar{A} \omega Z_0 (a - b)$$

$$F4 = - i 2\bar{B} \omega Z_0 \int_0^{2\pi} (a \cos \eta + i b \sin \eta)(\cos 2\eta + i \sin 2\eta) d\eta$$

$$= - i 2\bar{B} \omega Z_0 (o)$$

$$= 0$$

$$T = - \frac{1}{4} i \int \rho w d\bar{w}$$

$$= - \frac{1}{4} i \rho (2\pi i (A\bar{A} + 2 B\bar{B}) + 2\pi i \omega^2 Z_0^2 a b$$
$$+ \pi i \bar{A} \omega Z_0 (a - b)$$

In this case $A = \bar{A} = 0$ because there is no Z translation.

Therefore:

$$T = - \frac{1}{4} i \rho (2\pi i (2 B\bar{B}) + 2\pi i \omega^2 Z_0^2 a b)$$

$$= \frac{\rho\pi}{2} (2 \omega^2 (a^2 + b^2) + \omega^2 Z_0^2 a b)$$

for the plate, $b = 0$

so $T = \frac{\rho\pi}{16} \omega^2 a^4$

VERYI $= \frac{1}{8} \pi \rho a^4$ per unit width

$= \frac{1}{8} \pi \rho (\text{WETL})^4$ BEAM.

APPENDIX C.

From Figure 16 of (12) the stability of a planing craft can be increased by increasing

$$\sqrt{\frac{C_L}{2}} \text{ i.e. } C_L.$$

But $C_L = .0120\,\lambda^{1/2}\,\tau^{1.1}$ (page 10, (12))

Since increasing C_L increases stability, increasing λ increases stability. In this case $\lambda = \frac{l_m}{b}$. This means that a decrease in b will result in an increase in stability.

From Figure 20 of (2) the stability criterion is

$$\frac{C_{Lb}}{l_{cp}/b} = \frac{1.80}{(F_\nabla)^{2.5}}$$

where:

$$C_{Lb} = \frac{W}{\frac{1}{2}\rho V^2 b^2}$$

and:

$$F = \left(\frac{V}{g\nabla^{\frac{1}{3}}}\right)^{\frac{1}{2}}$$

If $\dfrac{C_{Lb}}{\frac{l_{cp}}{b}} = \dfrac{W}{\frac{1}{2}\rho V^2 b\ l_{cp}}$ decreases, the boat is stabilized. Therefore increasing b stabilizes the boat.

A comparison of the two methods makes it difficult to decide what effect b has on stability.

APPENDIX D.

In order to determine the resistance of a planing surface it is necessary to know the mean velocity over the bottom.

In Figure 12 of (12) a method is provided for determining this graphically. Part of an analytic solution is provided which, if completed, would be useful in the computation of mean velocity in a computer program.

The following is a resume of the development of the complete equation. From (12):

$$\frac{VM}{V} = \sqrt{1. - \frac{0.0120\ \tau^{1.1}}{\lambda^{\frac{1}{2}} \cos \tau}\ f(\beta)}$$

where:

VM = mean velocity over bottom (fps)

V = the u_0, the horizontal velocity of the origin of coordinates (fps)

τ = trim angle of planing surface (degrees)

$f(\beta)$ = an undetermined function.

By changing τ to radians and plotting

$$\left[1 - \left(\frac{VM}{V}\right)^2\right] \frac{\lambda^{\frac{1}{2}} \cos \tau}{.0120\ \tau^{1.1}} = f(\beta)$$

it was possible to find an $f(\beta)$ which yielded satisfactory results.

The final result is:

$$\frac{VM}{V} = \sqrt{1. - \frac{0.120\ \tau^{1.1}}{\lambda^{\frac{1}{2}} \cos \tau} \cdot \frac{80. - 50. \beta}{\cos^3 \beta}}$$

where:

$$f(\beta) = \frac{80. - 50. \beta}{\cos^3 \beta}$$

τ = trim angle in radians

β = deadrise angle in radians.

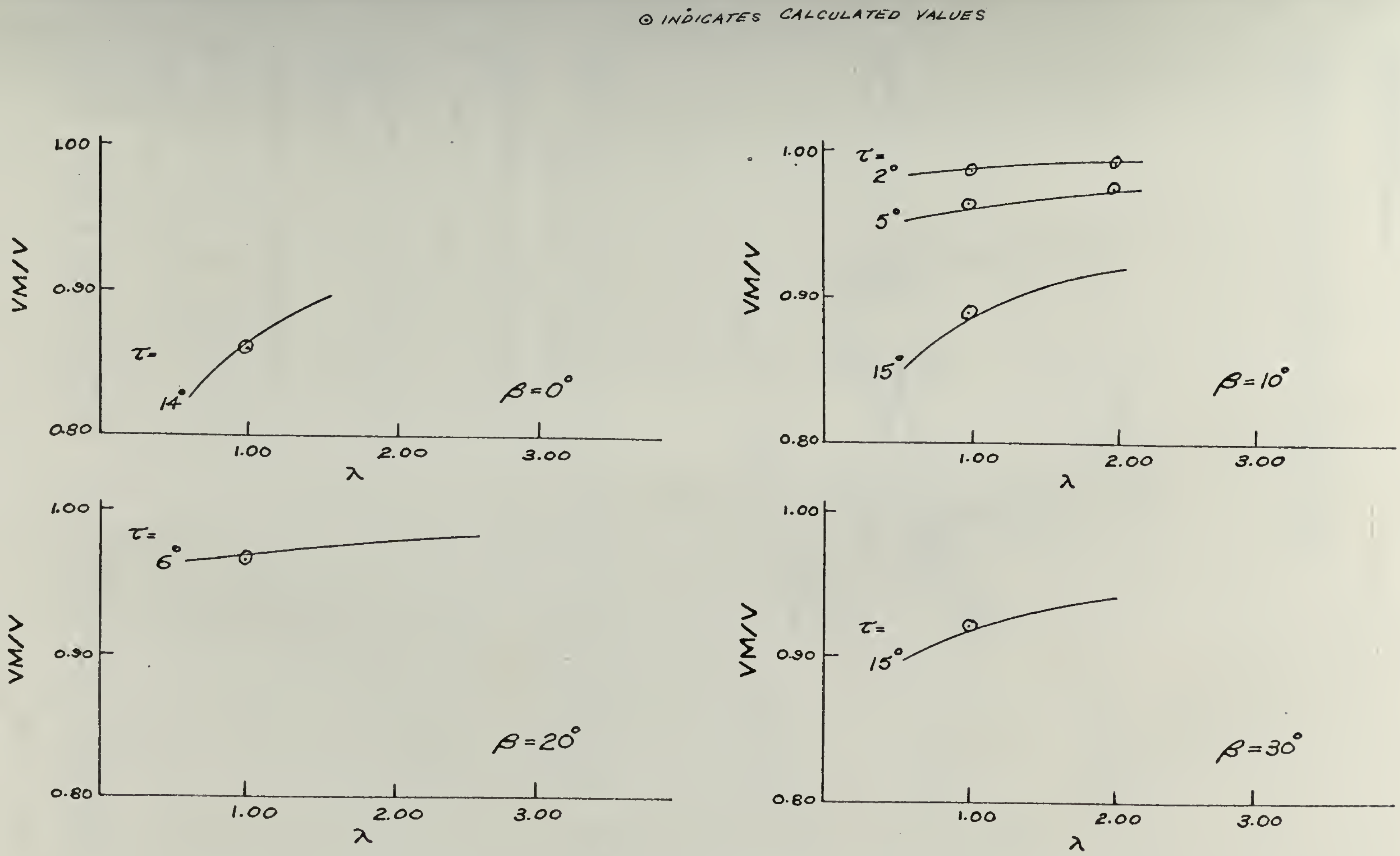


Figure 7.

VM/V vs. λ from Figure 12 of Reference 12 compared with Values Calculated using the Empirical Function f(β).

APPENDIX E.

EQUILIBRIUM PLANING CONDITIONS.

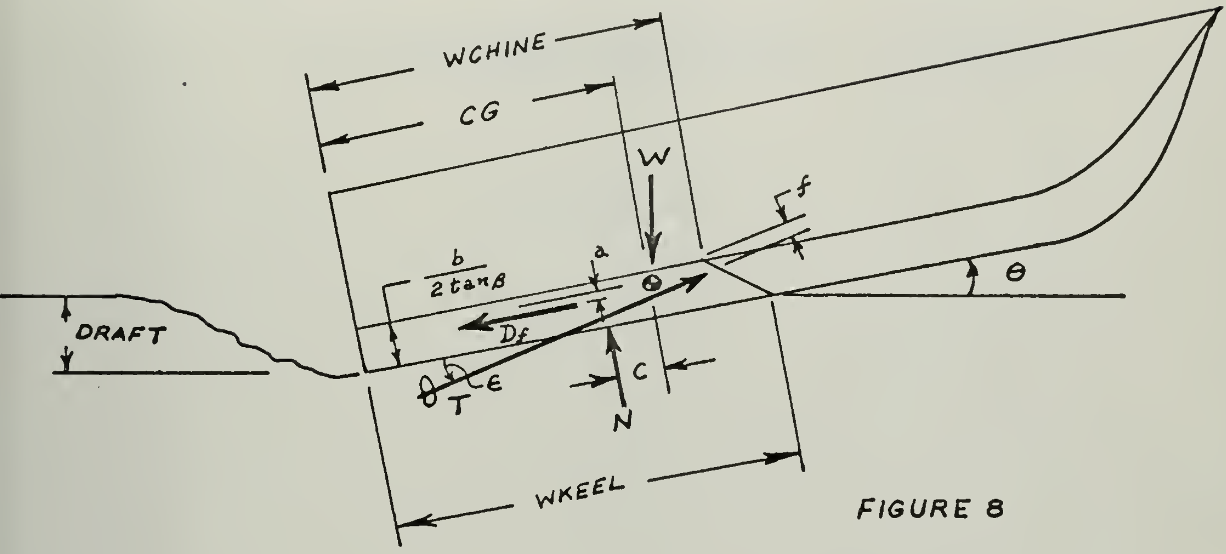


FIGURE 8

Note: This development follows reference (12) in general. However, the final result contains terms neglected by SAVITSKY.

Summation of forces in vertical direction:

$$(1)\quad W = N \cdot \cos\theta + T \cdot \sin(\theta + \epsilon) - D_f \cdot \sin\theta$$

Summation of pitching moments:

$$(2)\quad N \cdot c + D_f \cdot a - T \cdot f = 0$$

Summation of forces along keel:

$$(3)\quad T \cdot \cos\epsilon = D_f + W \cdot \sin\theta$$

Combining (1) and (3):

$$(4)\quad N = \frac{W}{\cos\theta \cdot \cos\epsilon}\left[\cos\epsilon - \sin\theta \cdot \sin(\theta + \epsilon)\right] + \frac{D_f}{\cos\theta . \cos\epsilon} \cdot \left[\sin\theta \cdot \cos\epsilon - \sin(\theta + \epsilon)\right]$$

Combining (2) and (3):

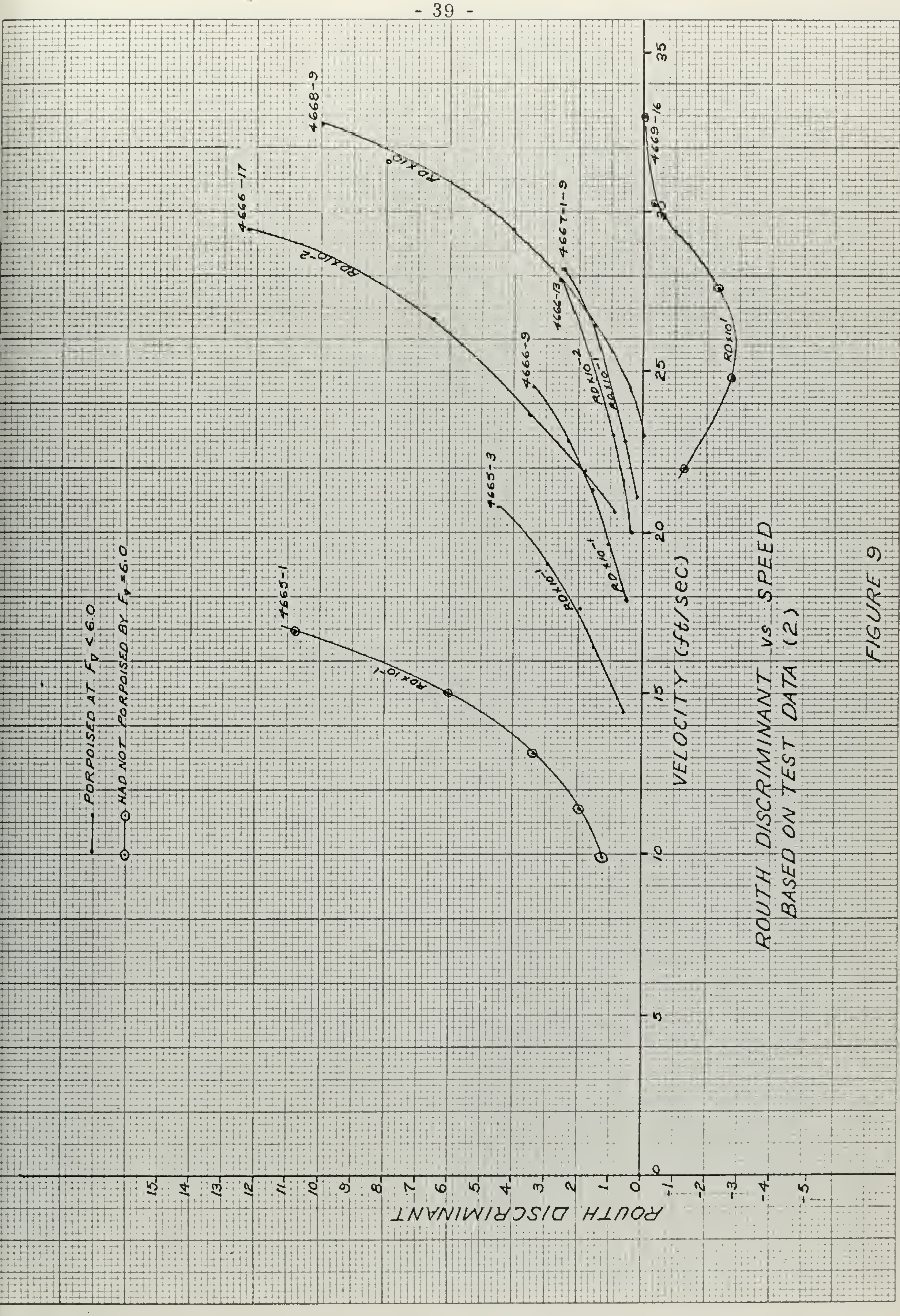
$$(5)\quad N \cdot c + D_f \cdot a - \frac{f}{\cos \epsilon} \left[W \cdot \sin \theta + D_f \right] = 0$$

Combining (4) and (5), we obtain the general equilibrium requirement:

$$W \left[\frac{\left[\cos \epsilon - \sin \theta \ \sin (\theta + \epsilon) \right] c}{\cos \theta \cdot \cos \epsilon} - \frac{f \cdot \sin \theta}{\cos \epsilon} \right]$$

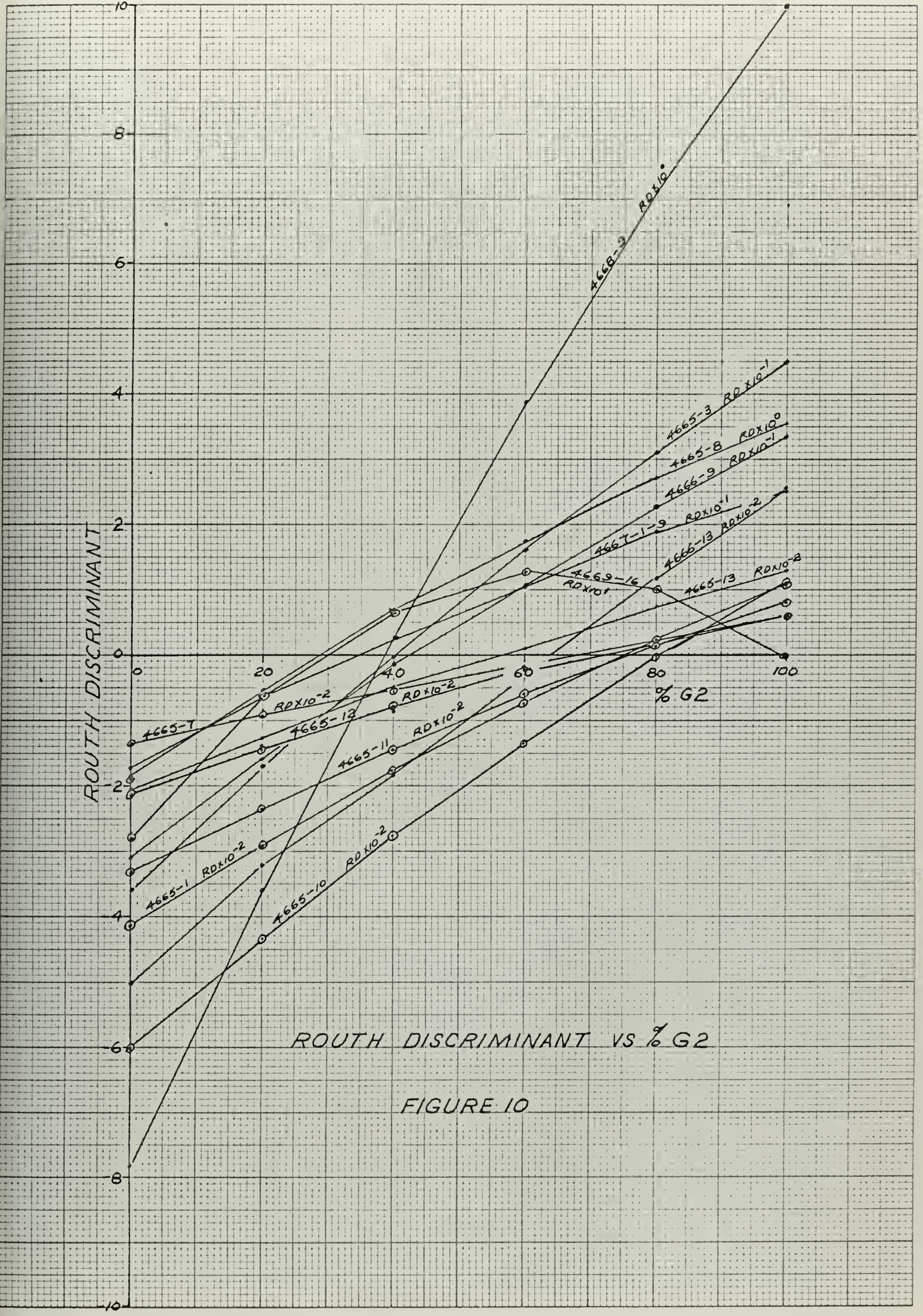
$$+ D_f \left[\frac{\left[\sin \theta \ \cos \epsilon - \sin (\theta + \epsilon) \right]}{\cos \theta \ \cos \epsilon} c + a - \frac{f}{\cos \epsilon} \right] = 0$$

APPENDIX F



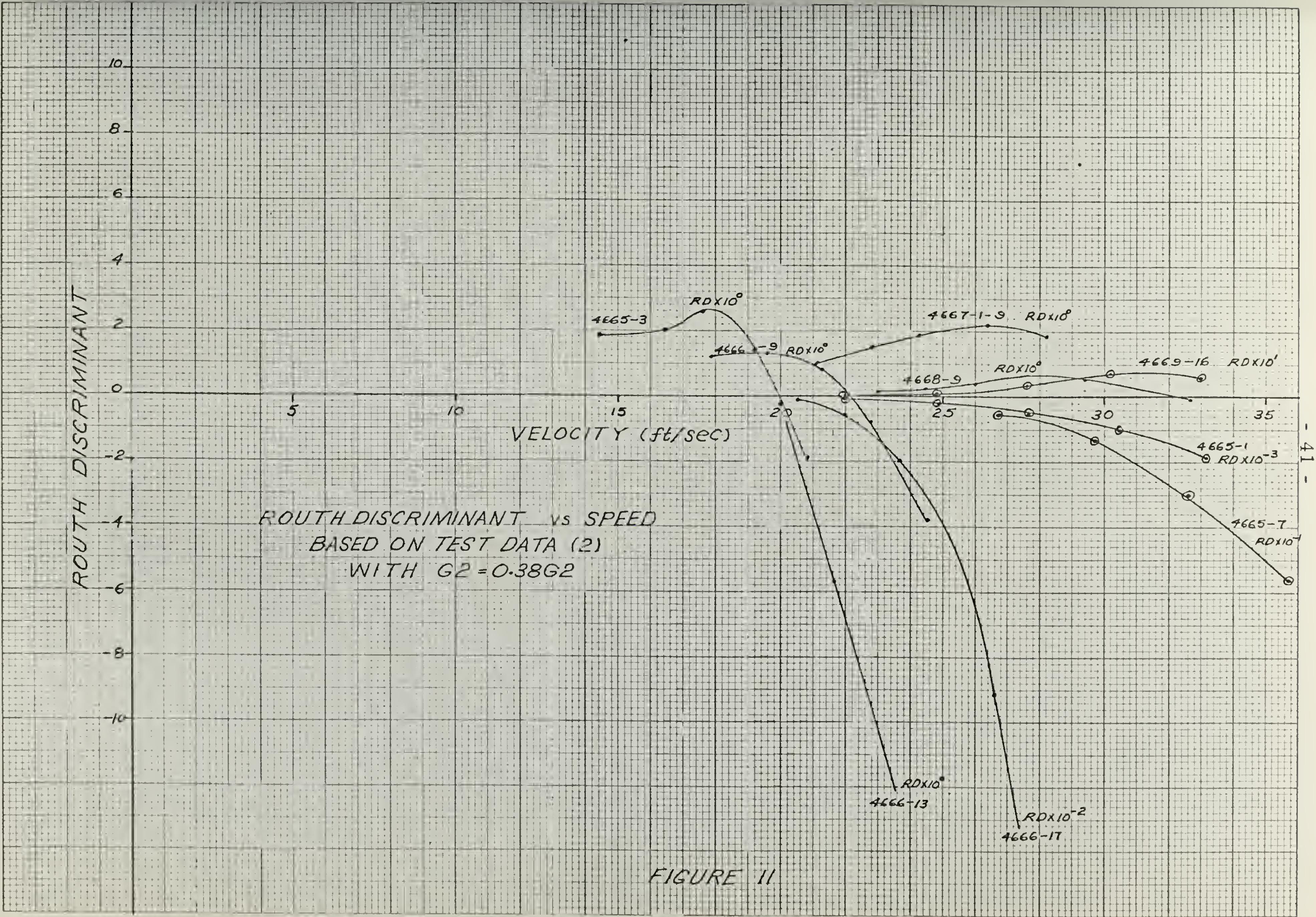
ROUTH DISCRIMINANT vs SPEED
BASED ON TEST DATA (2)

FIGURE 9



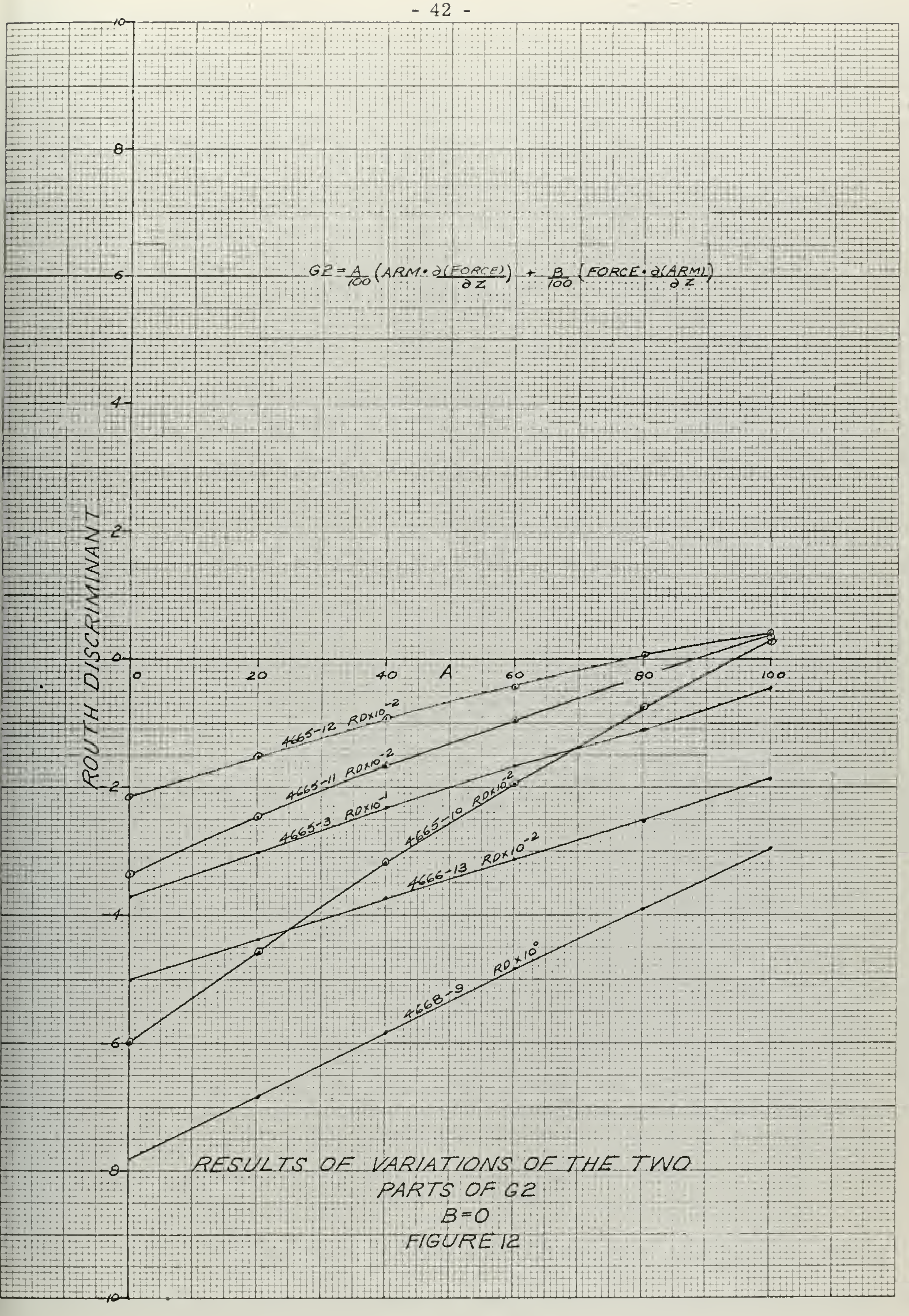
ROUTH DISCRIMINANT VS % G2

FIGURE 10



ROUTH DISCRIMINANT vs SPEED
BASED ON TEST DATA (2)
WITH G2 = 0.38G2

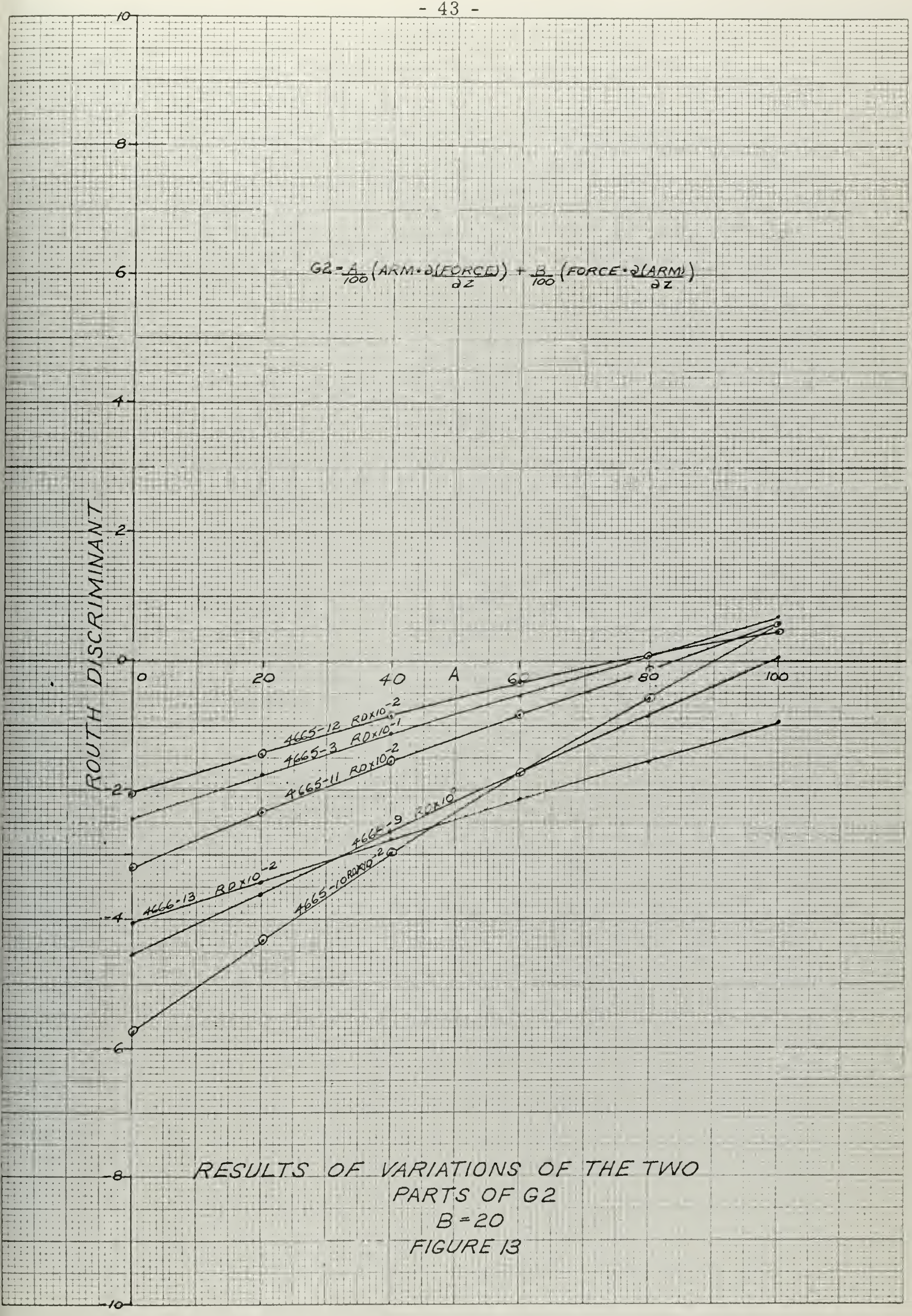
FIGURE 11



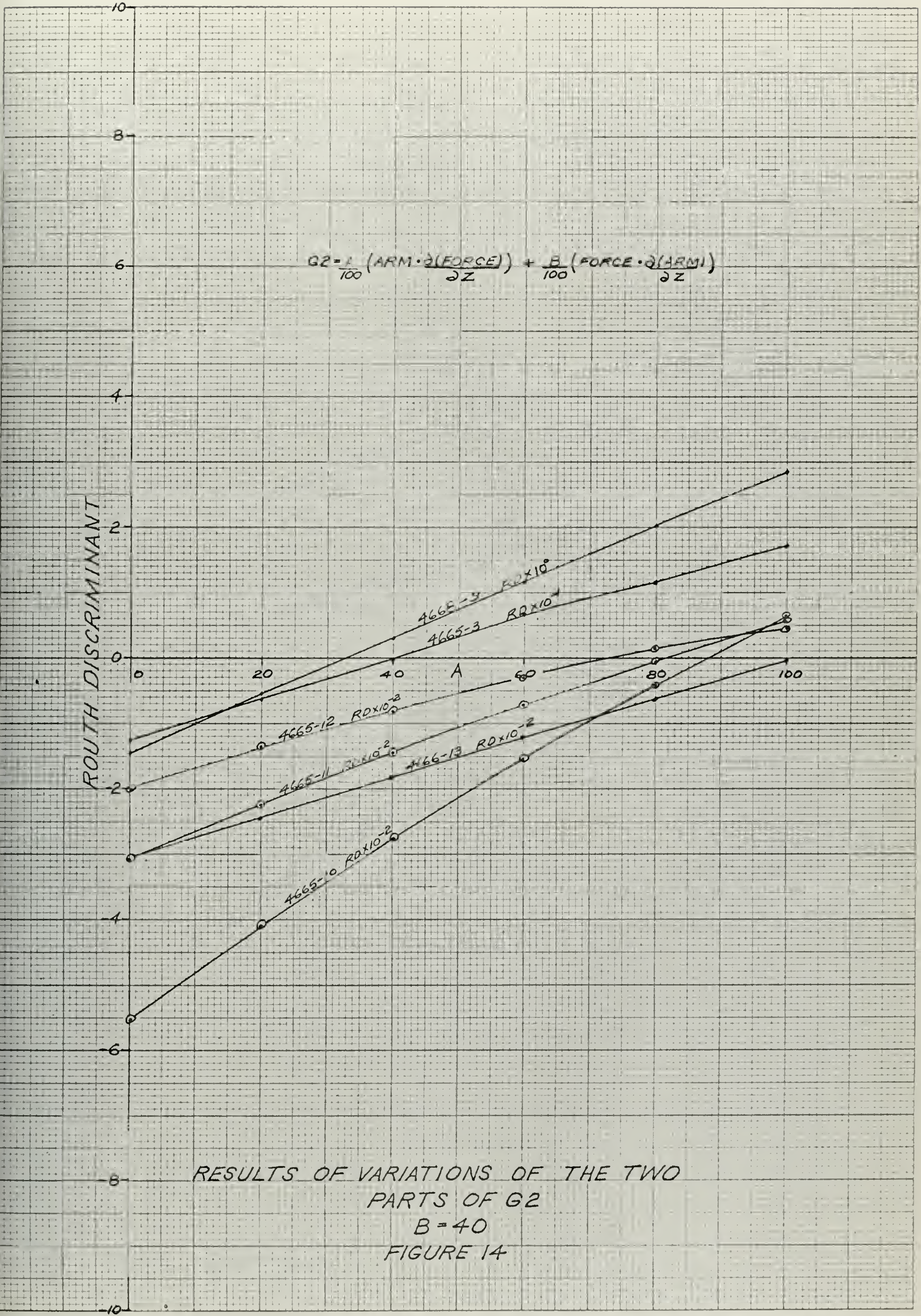
RESULTS OF VARIATIONS OF THE TWO PARTS OF G2

$B=0$

FIGURE 12

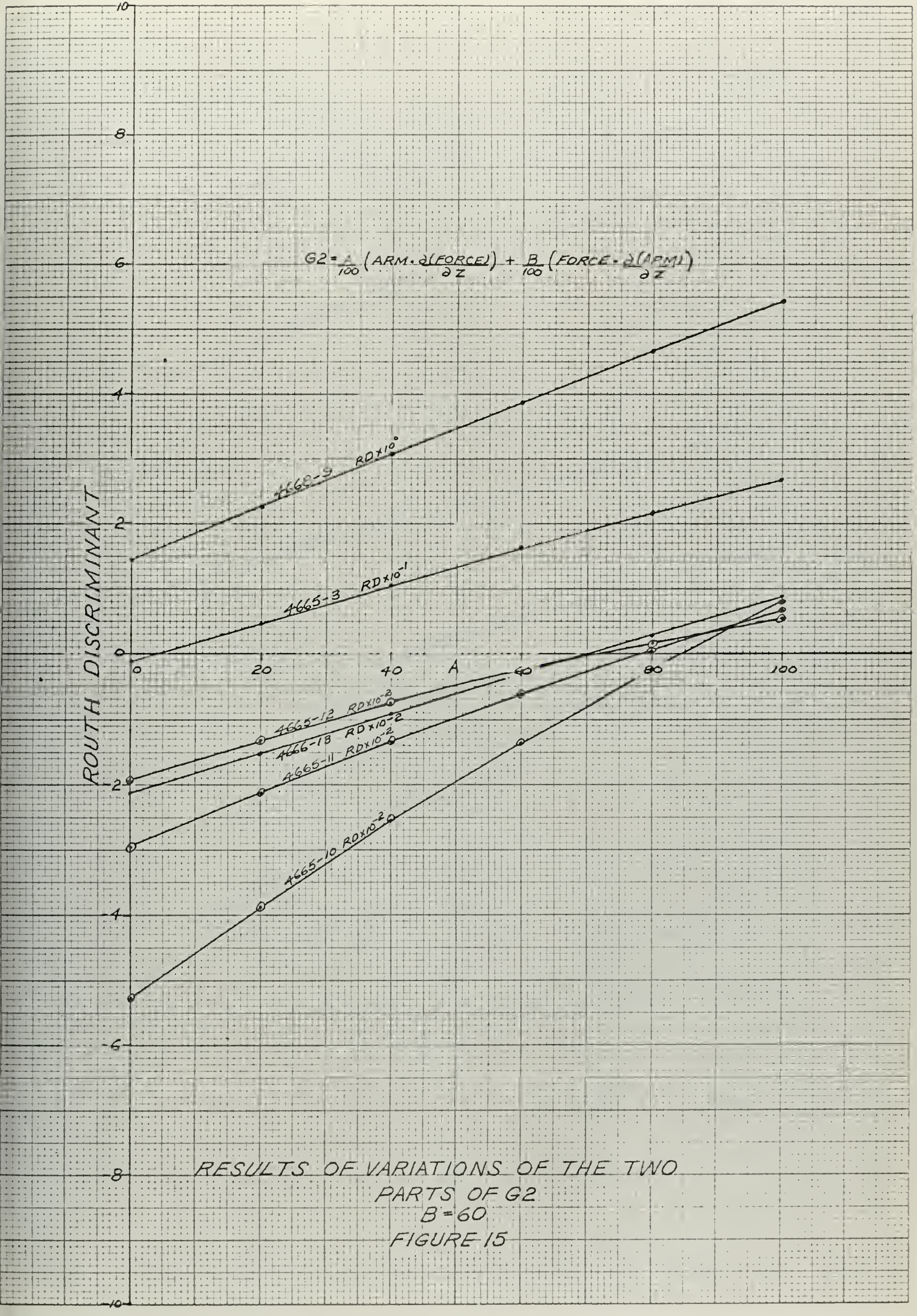


RESULTS OF VARIATIONS OF THE TWO
PARTS OF G2
B = 20
FIGURE 13

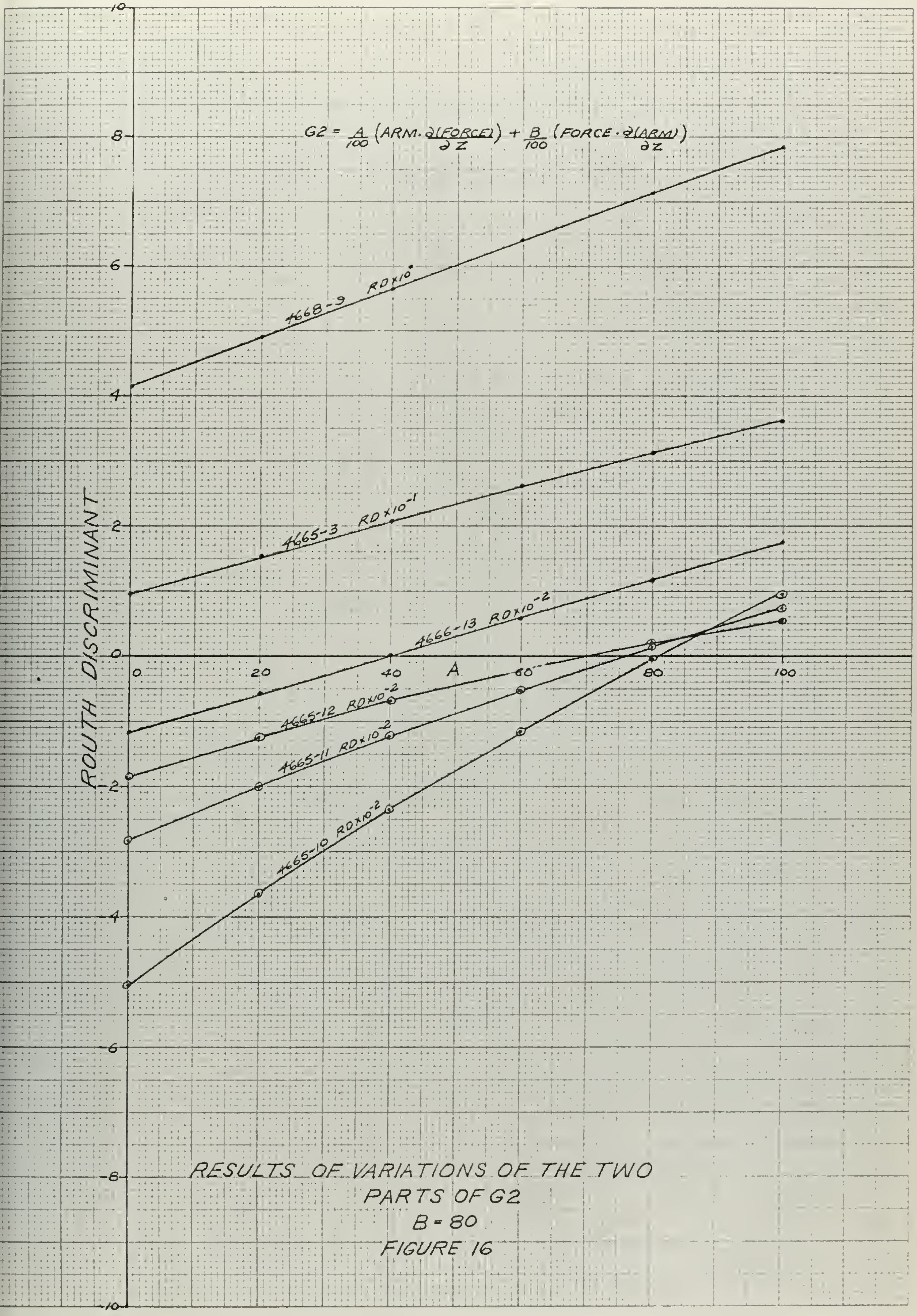


RESULTS OF VARIATIONS OF THE TWO PARTS OF G2
B = 40
FIGURE 14

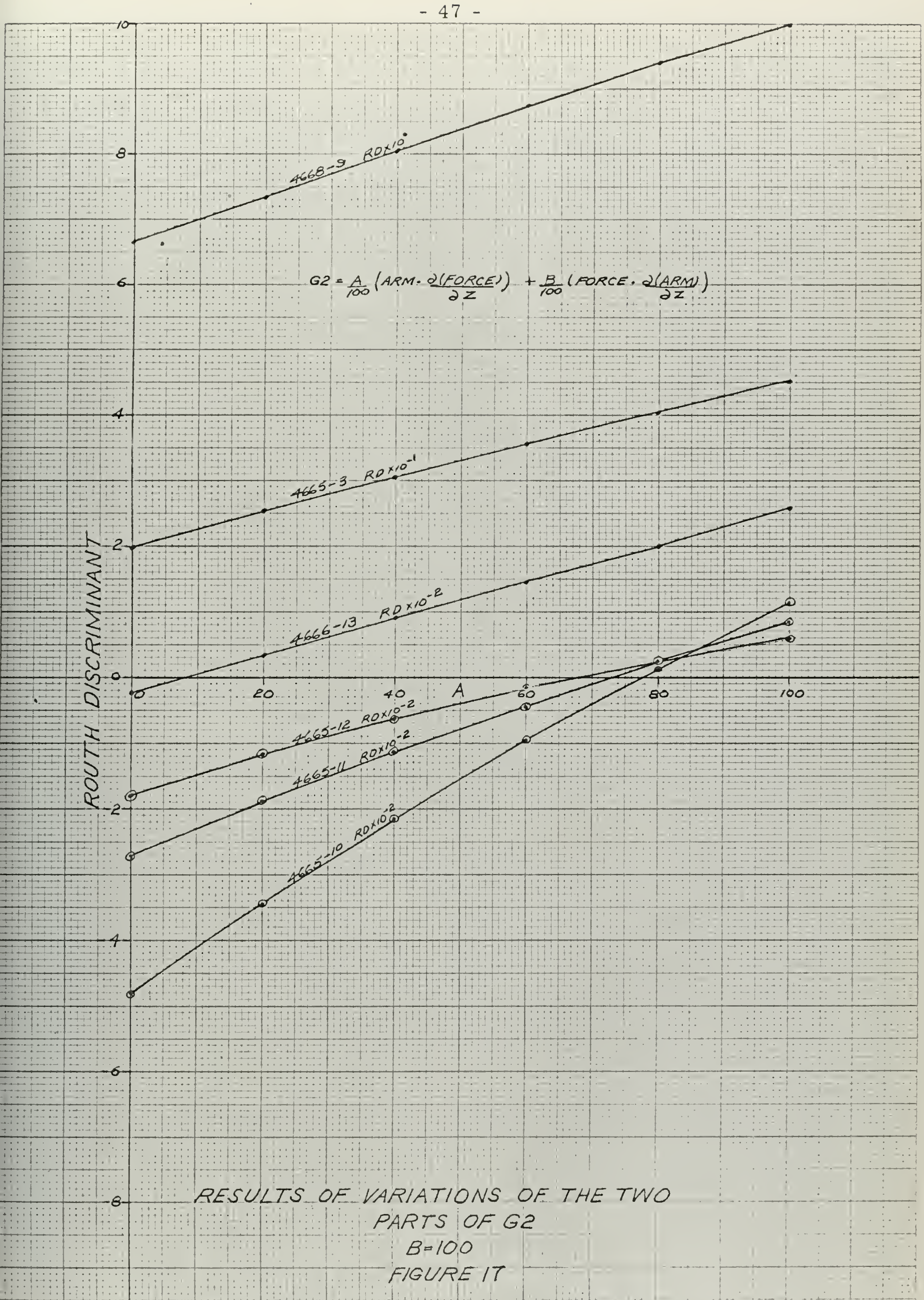
the Inch



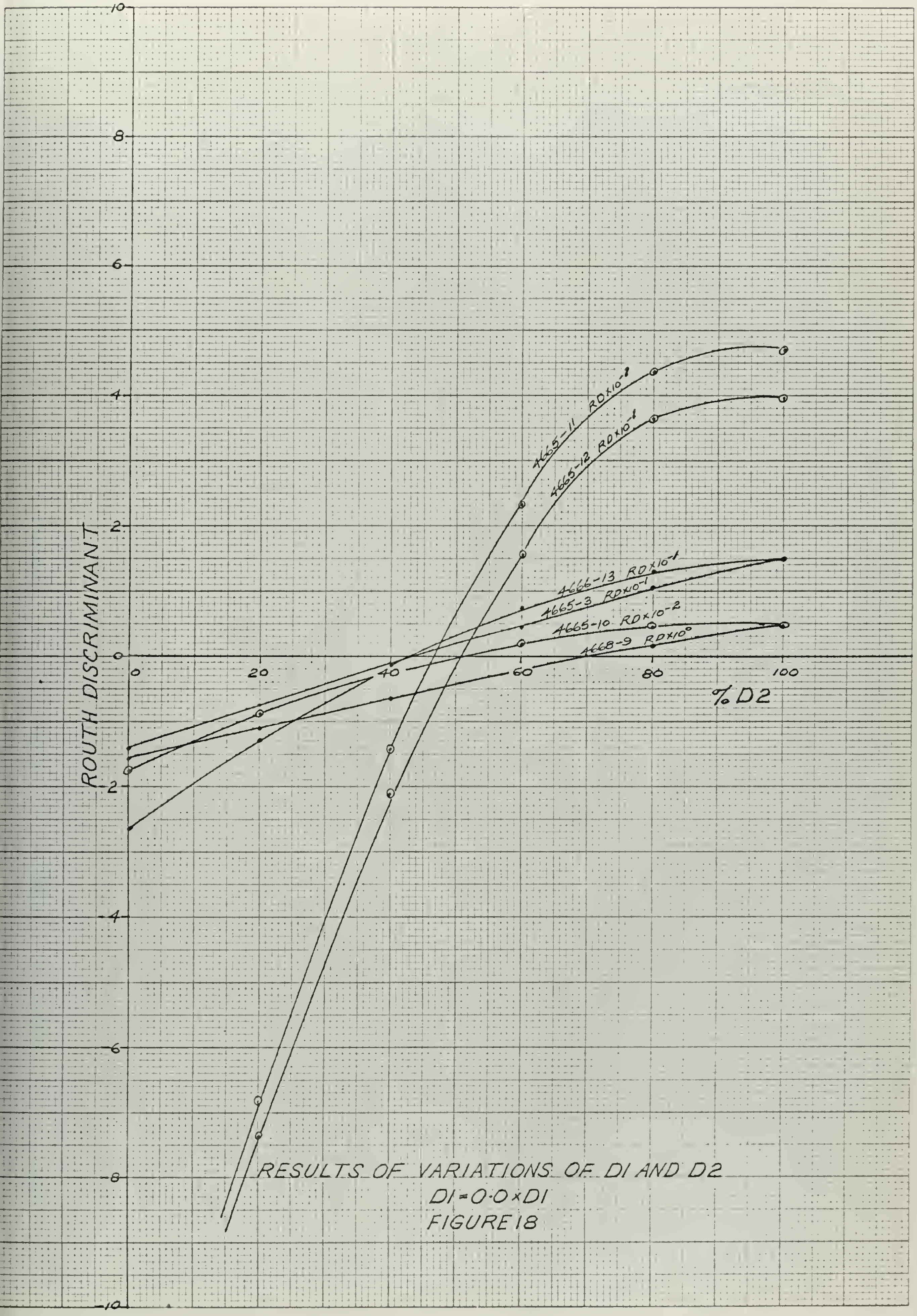
RESULTS OF VARIATIONS OF THE TWO
PARTS OF G2
B=60
FIGURE 15



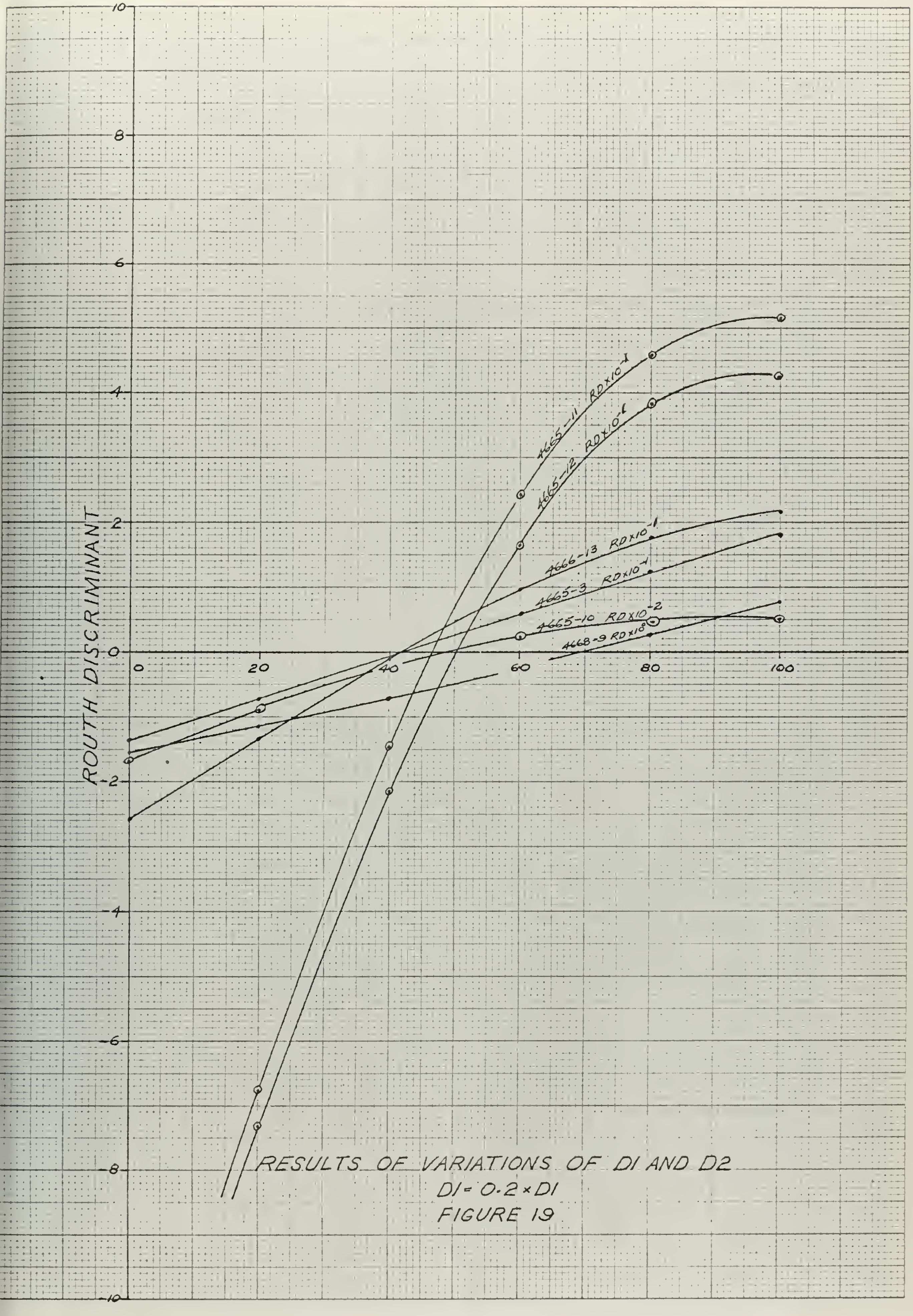
RESULTS OF VARIATIONS OF THE TWO
PARTS OF G2
B = 80
FIGURE 16



RESULTS OF VARIATIONS OF THE TWO
PARTS OF G2
B=100
FIGURE 17



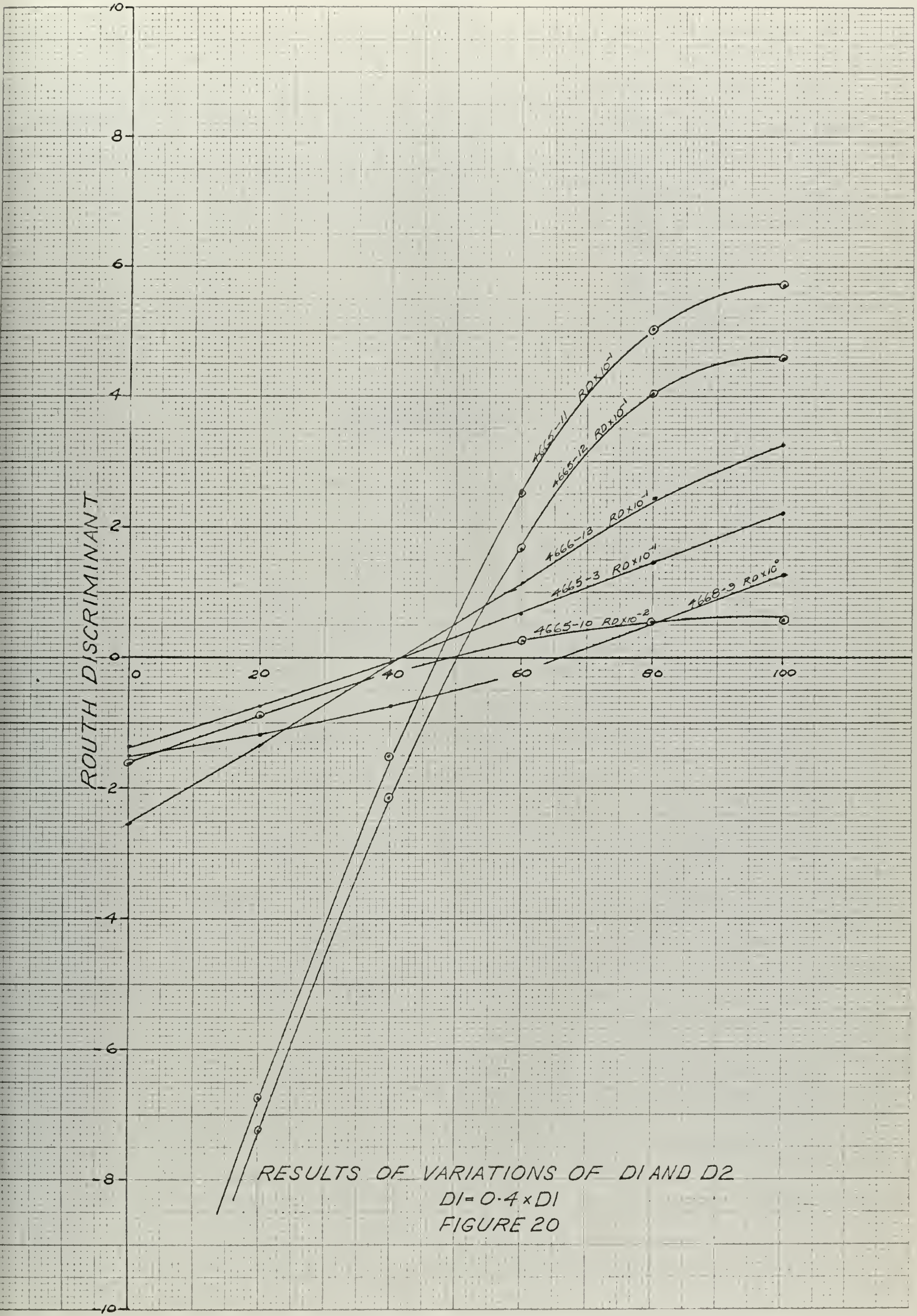
RESULTS OF VARIATIONS OF D1 AND D2
D1 = 0.0 × D1
FIGURE 18



RESULTS OF VARIATIONS OF D1 AND D2

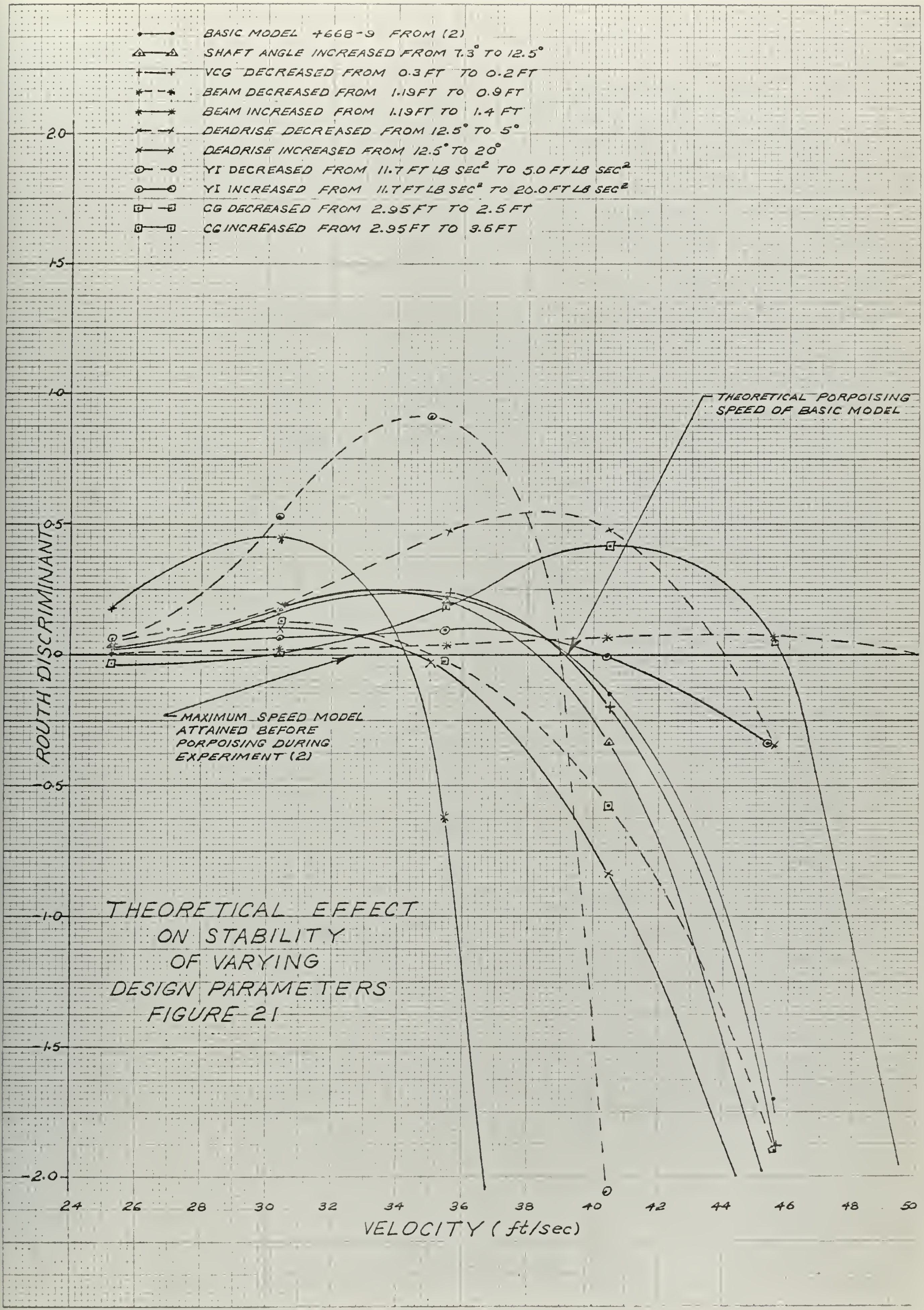
D1 = 0.2 × D1

FIGURE 19



RESULTS OF VARIATIONS OF D1 AND D2
D1= 0·4 × D1
FIGURE 20

APPENDIX G



THEORETICAL EFFECT ON STABILITY OF VARYING DESIGN PARAMETERS

FIGURE 21

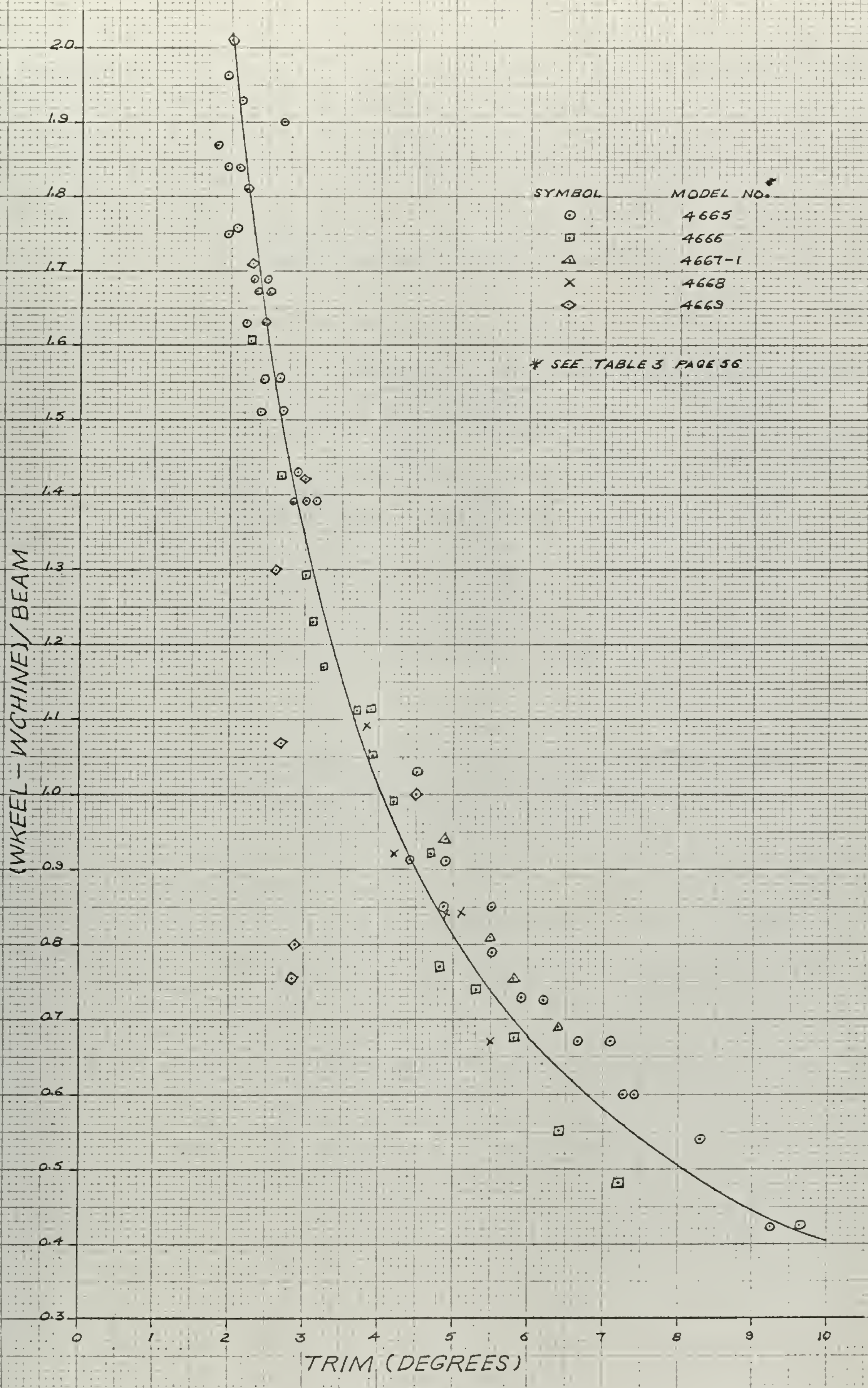

(WKEEL – WCHINE)/ BEAM
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0
1
2
3
4
5
6
7
8
9
10
TRIM (DEGREES)
SYMBOL
MODEL NO.*
⊙ 4665
⊡ 4666
◬ 4667-1
× 4668
◇ 4669
* SEE TABLE 3 PAGE 56

FIGURE 22

Experimental Results from (2)

	As computed from program 2	Experimental Results
Model No.	4668 AT 32.7 FPS	
Trim (degrees)	3.68	3.80
W Keel (ft.)	5.09	4.40
W Chine (ft.)	3.78	3.10
Drag (lbs.)	25.73	25.09

Table 1. Comparison of calculated and experimental hydrodynamic characteristics.

Data

For Model 4668 Test No. 9 from (2) for the 19.36 Knot Speed.

Derivative	As computed from experimental planing conditions	As computed from calculated planing conditions
A11	10.16	13.16
B11	3.13	3.30
C11	2.43	2.38
D11	6.68	6.32
E11	27.44	36.45
G11	20.44	21.55
A22	7.25	9.47
B22	22.61	20.58
C22	5.59	-1.06
D22	6.68	6.32
E22	0.89	-0.14
G22	1.20	1.20
RD	-0.071	0.209

Table 2. Comparison of coefficients of equations of motion as computed from experimental data and data generated from computer program 2.

Table 3. Model Data (2). All models have deadrise = 12.5°

Model Number	Run	Weight (lbs)	Beam (ft)	Length (ft)	CG (ft)	*F (at max. test speed)
4665	1	54.50	1.654	3.912	1.62	5.98
4665	3	129.08	1.654	3.912	1.70	3.24
4665	7	80.07	1.654	3.912	1.70	6.05
4665	8	80.07	1.654	3.912	1.55	3.50
4665	9	80.07	1.654	3.912	1.39	2.74
4665	10	55.77	1.654	3.912	1.86	5.96
4665	11	54.50	1.654	3.912	1.70	5.99
4665	12	54.50	1.654	3.912	1.55	5.98
4665	13	54.50	1.654	3.912	1.39	3.23
4666	9	146.20	1.623	5.987	2.17	3.75
4666	13	101.80	1.623	5.987	2.17	4.53
4666	17	76.10	1.623	5.987	2.17	5.01
4667-1	9	221.10	1.600	8.00	2.95	4.02
4668	9	141.80	1.190	8.00	2.95	5.03
4669	16	51.40	0.935	8.00	3.27	6.02

* Note: Unstable boats, as refered to in this paper, are those which porpoised at F less than 6.0. Stable boats are those which had not porpoised before maximum test speed of reference (2) was attained (F $\approx$ 6.0).

APPENDIX H

COMPUTER PROGRAMS

PROGRAM 1

Program 1 can be used to solve for the stability derivatives, coefficients of the equations of motion, the coefficients of the characteristic equation and the Routh discriminant using experimental data as input.

Data cards are punched in the manner indicated by READ 1 and 1 FORMAT where:

W	weight of boat in (lbs)
ALFAO	trim angle when boat is at rest (degrees)
CG	longitudinal position of center of gravity forward of transom (ft)
C	forward speed (knts)
RT	towing force (lbs)
WK	wetted length of keel (ft)
WC	wetted length of chine (ft)
S	wetted area (ft^2)
TRIM	(planing angle - α_o) (degrees) is change of trim from the at rest position
YI	moment of inertia about the Y-axis. Axis taken through center of gravity. (lb ft sec^2)
RHO	density of water (lb sec^2/ft)
U	arbitrary non dimensionalizing velocity (ft/sec)
BEAM	beam of boat (ft)
BETAI	deadrise angle (degrees)
VCG	height of center of gravity above the keel (ft)
EPSIL	shaft angle (degrees).

```
C                              COMPUTER PROGRAM ONE

C     EQUATIONS OF MOTION OF PLANING HULLS

C

C     RD=ROUTH DISCRIMINANT

C     AA,BB,ETC=ROUTH CRITERION FACTORS

C     A11,B11,ETC=NONDIMENSIONAL FORCE AND MOMENT COEFFICIENTS

C     A1,B1,ETC=FORCE AND MOMENT COEFFICIENTS

C     READ INPUT DATA

C

  666 READ1,W,ALFAO,CG,C,RT,WK,WC,S,TRIM,YI,RHO,U,BEAM,BETAI,
     1VCG,EPSIL

    1 FORMAT(F7.2,F5.2,F4.2,F5.2,F5.2,F4.2,F4.2,F5.2,F4.2,F5.2,
     1F5.3,F3.0,F5.3,F4.1,F4.2,F5.2)

      V=C*1.689

      WETL=(WK+WC)/2.0

      VERM=.125*RHO*3.1416*(WETL**2)*BEAM

      VERYI=.0625*.0625*RHO*3.1416*(WETL**4)*BEAM

      CV=V/SQRTF(32.2*BEAM)

      A=WETL/BEAM

      CPL=WETL*(0.75-1.0/(5.21*((CV/A)**2)+2.39))

      TAU=(ALFAO+TRIM)/57.2956

      BETA=BETAI/57.2956

      EPS=EPSIL/57.2956

C

C     DIFFERENTIATION OF LIFT COEFFICIENT X AREA WITH RESPECT

C     TO TAU
```

```
C
      TAU1=TAU-0.001
      TAU2=TAU+0.001
      A=1./A
      CLVOL1=1./(2.*WETL*(CV**2))*(((WC**2)*SINF(2.*TAU1)
     1/BEAM+1./3.*(2.*WC+WK)*SINF(BETA)/COSF(BETA)))
    3 CLB1=0.5*CLVOL1
      CLSA1=(1.5708*A*TAU1*(COSF(TAU1)**2)*(1.0-SINF(BETA))
     1/(1.0+A)+4.0*(SINF(TAU1)**2)*(COSF(TAU1)**3)*COSF(BETA)/
     23.0+CLB1)*S
      CLVOL2=1./(2.*WETL*(CV**2))*(((WC**2)*SINF(2.*TAU2)
     1/BEAM+1./3.*(2.*WC+WK)*SINF(BETA)/COSF(BETA)))
    5 CLB2=0.5*CLVOL2
      CLSA2=(1.5708*A*TAU2*(COSF(TAU2)**2)*(1.0-SINF(BETA))
     1/(1.0+A)+4.0*(SINF(TAU2)**2)*(COSF(TAU2)**3)*COSF(BETA)/
     23.0+CLB2)*S
      CL=1.5708*TAU*A*COSF(TAU)**2*(1.-SINF(BETA))/(1.+A)+4.*
     1SINF(TAU)**2*COSF(TAU)**3*COSF(BETA)/3.+(CLVOL1+CLVOL2)/4.
      TAO=TAU1
      TAB=TAU2
      CO=1.5708*TAU*A*COSF(TAO)**2*(1.-SINF(BETA))/(1.+A)+4.*
     1SINF(TAO)**2*COSF(TAO)**3*COSF(BETA)/3.+(CLVOL1)/2.
      CB=1.5708*TAU*A*COSF(TAB)**2*(1.-SINF(BETA))/(1.+A)+4.*
     1SINF(TAB)**2*COSF(TAB)**3*COSF(BETA)/3.+(CLVOL2)/2.
      DADTAU=1000.*W/(RHO*V**2)*(1./CB-1./CO)
      DIFB1=(CLSA2-CLSA1)/.002+CL*DADTAU
C
C     DIFFERENTIATION OF LIFT COEFF X AREA WITH RESPECT TO Z
```

```
C
      DELZ=0.001
      DELA=-DELZ*BEAM/SINF(TAU)/WETL**2
      DELS=BEAM*DELZ/(SINF(TAU)*COSF(BETA))
      DELL=DELZ/SINF(TAU)
      CLVOL3=(((WC-DELL)**2)*SINF(TAU)/BLAM+(2.*WC+WK-3.*DELL)
     1*SINF(BETA)/(COSF(BETA)*3.))/(2.*(WETL-DELL)*(CV**2))
    7 CLB3=0.5*CLVOL3
      CLSA3=(1.5708*(A-DELA)*TAU*(COSF(TAU)**2)*(1.-SINF(BETA))
     1/(1.+A-DELA)+4.*(SINF(TAU)**2)*(COSF(TAU)**3)*COSF(BETA)/3.
     2+CLB3)*(S-DELS)
      CLVOL4=(((WC+DELL)**2)*SINF(TAU)/BEAM+(2.*WC+WK+3.*DELL)
     1*SINF(BETA)/(COSF(BETA)*3.))/(2.*(WETL+DELL)*(CV**2))
    9 CLB4=0.5*CLVOL4
      CLSA4=(1.5708*(A+DELA)*TAU*(COSF(TAU)**2)*(1.-SINF(BETA))
     1/(1.+A+DELA)+4.*(SINF(TAU)**2)*(COSF(TAU)**3)*COSF(BETA)/3.
     2+CLB4)*(S+DELS)
      DIFC1 = (CLSA4-CLSA3)/.002
C
C     DIFFERENTIATION OF MOMENT WITH RESPECT TO Z
C
      CPL1=.75-(WETL-DELL)/(5.21*(CV*BEAM)**2/(WETL-DELL)**2+2.39)
      CPL2=.75-(WETL+DELL)/(5.21*(CV*BEAM)**2/(WETL+DELL)**2+2.39)
      DCPLDZ = (CPL2-CPL1)/.002
      C1=0.5*RHO*(V**2)*DIFC1
      DMDZ=(CG-CPL)*C1/COSF(TAU)-W*DCPLDZ/COSF(TAU)
      WTCL=.5*RHO*S*V**2*CL
      A1=W/32.2+VERM
```

```
      B1=0.5*RHO*V*DIFB1
      C1=0.5*RHO*(V**2)*DIFC1
      D1=VERM*VCG*(COSF(TAU)*(CG-.5*WETL)/VCG-SINF(TAU))
      E1=B1*(COSF(TAU)*(CG-WETL/2.)-VCG*SINF(TAU))
      G1=V*B1
      A2=YI+VERYI
      B2=E1*((CG-CPL)*COSF(TAU)-VCG*SINF(TAU))
      C2=G1*((CG-CPL)*COSF(TAU)-VCG*SINF(TAU))+W*(CPL*(SINF(TAU)-VCG*
     1COSF(TAU)/SINF(TAU)/WK*(COSF(TAU)/SINF(TAU)-1./SINF(TAU)))-CG*
     2SINF(TAU)-VCG*COSF(TAU))
      D2=VERM*(COSF(TAU)*(CG-WETL/2.)-VCG*SINF(TAU))
      E2=B1*((CG-CPL)*COSF(TAU)-VCG*SINF(TAU))
      G2 = DMDZ
      E1 = E1 + V*VERM
      G1=G1+V*B1
      B2 = B2 + V*D2
      C2=C2+V*E2

      QUANTITY 0.5 RHO CANCELLED OUT OF ALL FOLLOWING

      A11=A1/(BEAM**3)
      B11=B1/(U*(BEAM**2))
      C11=C1/(BEAM*(U**2))
      D11=D1/(BFAM**4)
      E11=E1/(U*(BEAM**3))
      G11=G1/((BEAM*U)**2)
      A22=A2/(BEAM**5)
      B22=B2/(U*(BEAM**4))
```

```
   C22=C2/((BEAM**3)*(U**2))
   D22=D2/(BEAM**4)
   E22=E2/(U*(BEAM**3))
   G22=G2/((BEAM*U)**2)
   AA=1.
   BB=(A22*B11+A11*B22-D22*E11-E22*D11)/(A11*A22-D11*D22)
   CC=(A22*C11+B22*B11+A11*C22-D22*G11-E22*E11-G22*D11)/(A11
  1*A22-D11*D22)
   DD=(B22*C11+B11*C22-E22*G11-G22*E11)/(A11*A22-D11*D22)
   EE=(C22*C11-G22*G11)/(A11*A22-D11*D22)
   RD=BB*CC*DD-AA*(DD**2)-(BB**2)*EE
   PRINT39
39 FORMAT(17X,3HA11,17X,3HB11,17X,3HC11,17X,3HD11,17X,3HE11,
  117X,3HG11)
   PRINT 40,A11,B11,C11,D11,E11,G11
40 FORMAT(6F20.5)
   PRINT 41
41 FORMAT(15X,3HA22,15X,3HB22,15X,3HC22,15X,3HD22,15X,3HE22,
  115X,3HG22,15X,3H RD)
   PRINT42,A22,B22,C22,D22,E22,G22,RD
42 FORMAT(7F18.5)
   PRINT43
43 FORMAT(14X,2HBB,14X,2HCC,14X,2HDD,14X,2HEE,13X,3HRD1,11X,
  15HDIFB1,11X,5HDIFC1)
   PRINT44,BB,CC,DD,EE,RD1,DIFB1,DIFC1
44 FORMAT(7F16.5)
   PRINT11,W,ALFAO,CG,V,RT,WK,WC,S,TRIM,YI,RHO,U,BEAM,BETAI,
  1VCG,EPSIL
```

```
11 FORMAT(F7.2,F5.2,F4.2,F5.2,F5.2,F4.2,F4.2,F5.2,F4.2,F5.2,
  1F5.3,F3.0,F5.3,F4.1,F4.2,F5.2//)
   GO TO 666
   END
```

PROGRAM 2.

This program solves for planing conditions: TRIM, ASPECT RATIO, WETTED KEEL, WETTED CHINE, WETTED AREA, DRAG, DRAFT at the transom, MEAN WETTED LENGTH, ESTIMATED EHP and the STABILITY INDICATOR (Routh discriminant: positive indicators imply stability, negative indicators imply instability. This section of the program does not yield satisfactory results as yet.)

A listing of all iterations involved in solving for the planing conditions and a listing of the coefficients of the equations of motion can be obtained as indicated by comments on the first page of the program print out.

This program uses as input, 1st card:

LIST 1, LIST 2, N BOATS

FORMAT (3 I 3).

LIST 1 and LIST 2 are as defined on first page of the program print out.

N BOATS is the number of different boats to be run.

2nd card:

BETAI, EPSILI, F, VCG, BEAM, CG, RHO, YI, W

FORMAT (4 F 5.2, 5 F 10.2)

BETAI = BETA of PROGRAM 1

EPSILI = EPSIL of PROGRAM 1

F is the perpendicular distance from shaft center line to CG (ft). All other variables are as defined on page 58.

3rd card:

NUMBER, IDENT

FORMAT (I 3, 5A4)

NUMBER = number of speed cards which are to follow

IDENT any identifying statement or symbol not to exceed 20 spaces.

4th card:

VKTS

FORMAT (F 10.2)

VKTS is velocity of boat in knots. One card is required for each speed. Number of cards equals NUMBER on card 3.

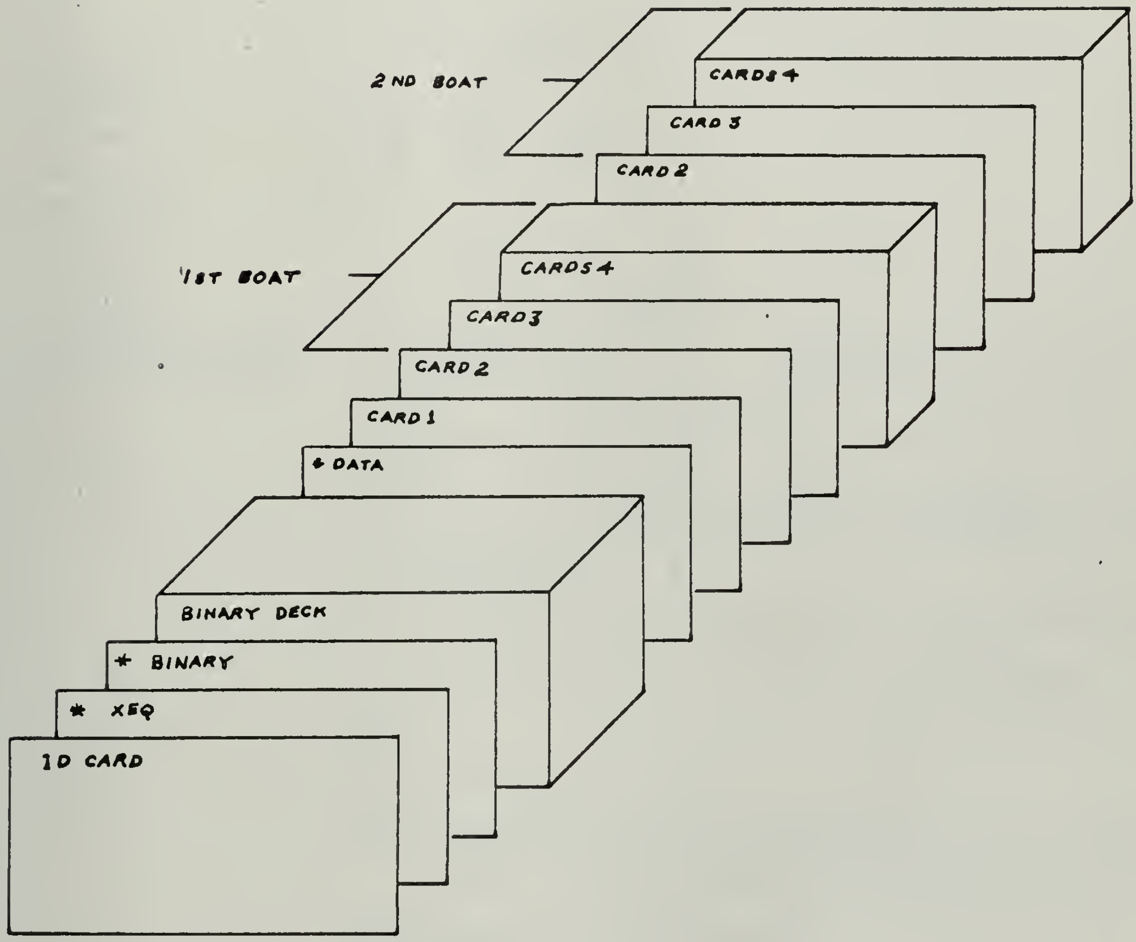


Figure 23. Program Assembly.

```
STABILITY CHARACTERISTICS FOR PLANING BOAT SERIAL  6809

EQUILIBRIUM PLANING CONDITIONS

VELOCITY (FPS) = 25.33
TRIM (DEG.) = 4.275
ASPECT RATIO = 4.03
WETTED KEEL (FT) = 5.02                 WETTED CHINE (FT) = 4.58
WETTED AREA (FT**2) = 5.73         DRAG (LBS)  = 21.40
DRAFT (FT) = .37
MEAN WETTED LENGTH (FT) = 4.80

ESTIMATED EHP =                      .99

STABILITY INDICATOR =        .13479E-01
```

Figure 25. Sample of Computer Output when both LIST 1 and LIST 2 equal 2.

```
C                      COMPUTER PROGRAM TWO
C     STABILITY OF PLANING CRAFT
      DIMENSION T1(15), VALUE(15),IDENT(5)
      COMMON AA, A11, A22, ASP, BB, B11, B22, BEAM, BETA, CC,
     1C11, C22, CLB, CLO, CG, CV, DD, D11, D22, DIFB1, DIFC1,
     2DRAG, EE, E11, E22, EPSIL, F, G11, G22, LIST1, LIST2, NR,
     3RHO, S, TAU, TRIM, T1, VALUE, V, VCG, VKTS, VM, W, WETL,
     4WCHINE, WKEEL, YI
      READ 1,LIST1, LIST2,NBOATS
C
C     IF LIST1= 1, PRINT OUT ALL COEFFICIENTS AND DERIVITIVES
C     ASSOCIATED WITH STABILITY EQUATIONS
C     IF LIST 1 = 2, PRINT OUT ONLY STABILITY INDICATOR
C     IF LIST 2 = 1, PRINT OUT ALL ITERATIONS INVOLVED IN
C     SOLVING FOR EQUILIBRIUM PLANING CONDITIONS
C     IF LIST 2 = 2, PRINT OUT ONLY FINAL PLANING CONDITIONS
C
    1 FORMAT (3I3)
      DO 7 NN=1,NBOATS
      READ2,BETAI,EPSILI,F,VCG,BEAM,CG,RHO,YI,W
    2 FORMAT(4F5.2,5F10.2)
      BETA=BETAI/57.2956
      EPSIL = EPSILI/57.2956
      READ 3,NUMBER,(IDENT(I),I=1,5)
    3 FORMAT(I3,5A4)
      PRINT101,(IDENT(I),I=1,5)
```

```
      DO 7 M = 1,NUMBER
      READ 4, VKTS
      V = VKTS* 1.689
    4 FORMAT(F10.2)
      CALL ANGLES
      CALL COEFF1
      EHP=V*DRAG/550.
      PRINT 116,EHP
      RD=BB*CC*DD-AA*DD**2-BB**2*EE
      IF(LIST1-1)6,5,6
    5 PRINT102,AA,BB,CC,DD,EE
      PRINT103,A11
      PRINT104,B11
      PRINT105,C11
      PRINT106,D11
      PRINT107,E11
      PRINT108,G11
      PRINT109,A22
      PRINT110,B22
      PRINT111,C22
      PRINT112,D22
      PRINT113,E22
      PRINT114,G22
    6 PRINT115,RD
    7 CONTINUE
  101 FORMAT(43H1STABILITY CHARACTERISTICS FOR PLANING BOAT,
     19H SERIAL    ,5A4///)
  102 FORMAT(46H THE COEFFICIENTS OF THE FOURTH-ORDER EQUATION,
```

```
    153H OF MOTION (AA*S**4 + BB*S**3 + CC*S**2 + DD*S + EE =
    2,33H 0.0) ARE AS FOLLOWS (AA THRU EE)/5F25.4//)
 103 FORMAT(46H FORCE DERIVITIVE FOR A UNIT INCREMENT OF VERT,
    119HICAL ACCELERATION =,F10.3)
 104 FORMAT(46H FORCE DERIVITIVE FOR A UNIT INCREMENT OF VERT,
    119HICAL VELOCITY     =,F10.3)
 105 FORMAT(46H FORCE DERIVITIVE FOR A UNIT INCREMENT OF VERT,
    119HICAL POSITION     =,F10.3)
 106 FORMAT(46H FORCE DERIVITIVE FOR A UNIT INCREMENT OF ANGU,
    119HLAR ACCELERATION  =,F10.3)
 107 FORMAT(46H FORCE DERIVITIVE FOR A UNIT INCREMENT OF ANGU,
    119HLAR VELOCITY      =,F10.3)
 108 FORMAT(46H FORCE DERIVITIVE FOR A UNIT INCREMENT OF ANGU,
    119HLAR POSITION      =,F10.3)
 109 FORMAT(47H MOMENT DERIVITIVE FOR A UNIT INCREMENT OF ANGU,
    119HLAR ACCELERATION  =,F10.3)
110  FORMAT(47H MOMENT DERIVITIVE FOR A UNIT INCREMENT OF ANGU,
    119HLAR VELOCITY      =,F10.3)
111  FORMAT(47H MOMENT DERIVITIVE FOR A UNIT INCREMENT OF ANGU,
    119HLAR POSITION      =,F10.3)
 112 FORMAT(47H MOMENT DERIVITIVE FOR A UNIT INCREMENT OF VERT,
    119HICAL ACCELERATION =,F10.3)
 113 FORMAT(47H MOMENT DERIVITIVE FOR A UNIT INCREMENT OF VERT,
    119HICAL VFLOCITY     =,F10.3)
 114 FORMAT(47H MOMENT DERIVITIVE FOR A UNIT INCREMENT OF VERT,
    119HICAL POSITION     =,F10.3)
 115 FORMAT(22H STABILITY INDICATOR =,E15.5////)
 116 FORMAT(16H ESTIMATED EHP =,F20.2///)
```

```
CALL EXIT
END
```

```
      SUBROUTINE ANGLES

      DIMENSION T1(15),VALUE(15)
      COMMON AA, A11, A22, ASP, BB, B11, B22, BEAM, BETA, CC,
     1C11, C22, CLB, CLO, CG, CV, DD, D11, D22, DIFB1, DIFC1,
     2DRAG, EE, E11, E22, EPSIL, F, G11, G22, LIST1, LIST2, NR,
     3RHO, S, TAU, TRIM, T1, VALUE, V, VCG, VKTS, VM, W, WETL,
     4WCHINE, WKEEL, YI
      CV=V/SQRTF(32.2*BEAM)
      CLB=2.*W/(RHO*((V*BEAM)**2))
      BETAI=BETA*57.2956
      ICLO=1
    5 CLO1=ICLO
      CLO=CLO1*0.01
      IF(CLO-0.0065*BETAI*CLO**.6-CLB)2,3,4
    2 ICLO=ICLO+1
      GO TO 5
    4 CLO=CLO-.001
      IF(CLO-0.0065*BETAI*CLO**.6-CLB)3,3,6
    6 GO TO 4
    3 CLO=CLO
      N=1
      NR = N
      T1(N)=1.
      IF(LIST2-1)11,1,11
    1 PRINT 1000,CLB,CLO,T1(N),N
   11 CALL FACTOR
      IF(ABSF(VALUE(N))-.0001)10,10,7
```

```
    7 N=2
      NR = N
      T1(N)=2.0
      IF(LIST2-1)12,13,12
   13 PRINT 1000,CLB,CLO,T1(N),N
   12 CALL FACTOR
      IF(ABSF(VALUE(N))-.0001)10,10,8
    8 CONTINUE
      DO 9 N = 3,8
      NR = N
      SLOPE = (VALUE(N-1) - VALUE(N-2))/(T1(N-1) -T1(N-2))
      T1(N) = T1(N-1) - VALUE(N-1)/SLOPE
      IF(LIST2-1)15,14,15
   14 PRINT 1000,CLB,CLO,T1(N),N
   15 CALL FACTOR
      IF(ABSF(VALUE(N))-.0001)10,10,9
    9 CONTINUE
   10 TERM=.5*SINF(BETA)*COSF(TAU)/3.1416/SINF(TAU)/COSF(BETA)
      WKEEL=BEAM*(ASP+TERM)
      WCHINE=BEAM*(ASP-TERM)
      DRAFT=WKEEL*SINF(TAU)
      WETL=ASP*BEAM
      TRIM=TAU*57.2956
      S=WETL*BEAM/COSF(BETA)
      PRINT 1002
      PRINT 1003,V,TRIM,ASP,WKEEL,WCHINE,S,DRAG,DRAFT,WETL
 1000 FORMAT(26H CALLING FACTOR WITH CLB =,F9.4,3X,7H ,CLO =,
     1F9.4,3X,13H ,AND  TRIM =,F20.4,15X,4H N =,I3)
```

```
1002 FORMAT(31H EQUILIBRIUM PLANING CONDITIONS//)
1003 FORMAT(17H VELOCITY (FPS) =,F7.2/14H TRIM (DEG.) =,F6.2/
    115H ASPECT RATIO =,F5.2/19H WETTED KEEL (FT) =,F6.2,20X,
    220H WETTED CHINE (FT) =,F6.2/22H WETTED AREA (FT**2) =,
    3F7.2,12X,13H DRAG (LBS) =,F7.2/13H DRAFT (FT) =,F5.2/
    426H MEAN WETTED LENGTH (FT) =,F6.2///)
     RETURN
     END
```

```
      SUBROUTINE FACTOR

      DIMENSION T1(15), VALUE(15)
      COMMON AA, A11, A22, ASP, BB, B11, B22, BEAM, BETA, CC,
     1C11, C22, CLB, CLO, CG, CV, DD, D11, D22, DIFB1, DIFC1,
     2DRAG, EE, E11, E22, EPSIL, F, G11, G22, LIST1, LIST2, NR,
     3RHO, S, TAU, TRIM, T1, VALUE, V, VCG, VKTS, VM, W, WETL,
     4WCHINE, WKEEL, YI
      N = NR
      IF (T1(N))2,2,3
    2 IF(LIST2-1)22,4,22
    4 PRINT 100,N,T1(N)
   22 T1(N) = .5
    3 ASP = 80.*(CLO/T1(N)**1.1)
      TAU=T1(N)/57.2956
      IF((.012*SQRTF(ASP)+.0055*ASP**2.5/CV**2)*T1(N)**1.1-CLO)
     111,8,12
   11 ASP=ASP+.01
      IF((.012* SQRTF(ASP)+.0055*ASP**2.5/CV**2)*T1(N)**1.1-CLO)
     111,8,8
   12 ASP = ASP - .01
      IF((.012*SQRTF(ASP)+.0055*ASP**2.5/CV**2)*T1(N)**1.1-CLO)
     18,8,12
    8 C=CG-(.75-1./(5.21*(CV/ASP)**2+2.39))*ASP*BEAM
      VM = V*SQRTF(1.-.012*TAU**1.1/SQRTF(ASP)*(85.-50.*BETA)/
     1COSF(BETA)**2/COSF(TAU))
      RE=131770.*ASP*BEAM*V
      REE=.43429448*LOGF(RE)
```

```
      CF=.008179
      DEL=.001
   50 CF=CF-DEL
      FRE=.242/SQRTF(CF)-.4329448*LOGF(CF)
      IF(REE-FRE)55,95,50
   55 IF(DEL-.001)65,60,60
   60 CF=CF+DEL
      DEL=.0001
      GO TO 50
   65 IF(DEL-.0001)75,70,70
   70 CF=CF+DEL
      DEL=.00001
      GO TO 50
   75 IF(DEL-.00001)85,80,80
   80 CF=CF+DEL
      DEL=.000001
      GO TO 50
   85 IF(DEL-.000001)95,90,90
   90 CF=CF+DEL
      DEL=.0000001
      GO TO 50
   95 CFT=CF+.0004
      A=VCG-.25*BEAM*SINF(BETA)/COSF(BETA)
      DRAG =RHO*(VM*BEAM)**2*ASP*CFT*.5/COSF(BETA)/COSF(TAU) +
     1W*SINF(TAU)/COSF(TAU)
      VALUE(N)=W*(C*(COSF(EPSIL)-SINF(TAU)*SINF(TAU+EPSIL))-F*
     1SINF(TAU)*COSF(TAU))+DRAG*(C*(SINF(TAU)*COSF(EPSIL)-
     2SINF(TAU+EPSIL))+(A*COSF(EPSIL)-F)*COSF(TAU))
```

```
      IF(LIST2-1)6,5,6
    5 PRINT 101,VALUE(N),CFT
  100 FORMAT(47H NEGATIVE ANGLE ENCOUNTERED ON ITERATION NUMBER,
     1I3,40H THE VALUE OF THIS ANGLE (IN DEGREES) IS,F10.3)
  101 FORMAT(32H RETURNING TO ANGLES WITH VALUE=,E12.5,
     110H AND CFT =,E12.5///)
    6 RETURN
      END
```

```
      SUBROUTINE COEFF1

      DIMENSION T1(15), VALUE(15)
      COMMON AA, A11, A22, ASP, BB, B11, B22, BEAM, BETA, CC,
     1C11, C22, CLB, CLO, CG, CV, DD, D11, D22, DIFB1, DIFC1,
     2DRAG, EE, E11, E22, EPSIL, F, G11, G22, LIST1, LIST2, NR,
     3RHO, S, TAU, TRIM, T1, VALUE, V, VCG, VKTS, VM, W, WETL,
     4WCHINE, WKEEL, YI
C
C     EQUATIONS OF MOTION OF PLANING HULLS
C     AA,BB,ETC=ROUTH CRITERION FACTORS
C     A11,B11,ETC=NONDIMENSIONAL FORCE AND MOMENT COEFFICIENTS
C     A1,B1,ETC=FORCE AND MOMENT COEFFICIENTS
C
      EPS = EPSIL
      A = ASP
      VERM=.125*RHO*3.1416*(WETL**2)*BEAM
      VERYI=.0625*.0625*RHO*3.1416*(WETL**4)*BEAM
      CPL=WETL*(0.75-1.0/(5.21*((CV/A)**2)+2.39))
C
C     DIFFERENTIATION OF LIFT COEFFICIENT X AREA WITH RESPECT
C     TO TAU
C
      TAU1=TAU-0.001
      TAU2=TAU+0.001
      WC = WCHINE
      WK = WKEEL
      A = 1./ASP
```

```
      CLVOL1=1./(2.*WETL*(CV**2))*(((WC**2)*SINF(2.*TAU1)
     1/BEAM+1./3.*(2.*WC+WK)*SINF(BETA)/COSF(BETA)))
    3 CLB1=0.5*CLVOL1
      CLSA1=(1.5708*A*TAU1*(COSF(TAU1)**2)*(1.0-SINF(BETA))
     1/(1.0+A)+4.0*(SINF(TAU1)**2)*(COSF(TAU1)**3)*COSF(BETA)/
     23.0+CLB1)*S
      CLVOL2=1./(2.*WETL*(CV**2))*(((WC**2)*SINF(2.*TAU2)
     1/BEAM+1./3.*(2.*WC+WK)*SINF(BETA)/COSF(BETA)))
    5 CLB2=0.5*CLVOL2
      CLSA2=(1.5708*A*TAU2*(COSF(TAU2)**2)*(1.0-SINF(BETA))
     1/(1.0+A)+4.0*(SINF(TAU2)**2)*(COSF(TAU2)**3)*COSF(BETA)/
     23.0+CLB2)*S
      CL=1.5708*TAU*A*COSF(TAU)**2*(1.-SINF(BETA))/(1.+A)+4.*
     1SINF(TAU)**2*COSF(TAU)**3*COSF(BETA)/3.+(CLVOL1+CLVOL2)/4.
      TAO=TAU1
      TAB=TAU2
      CO=1.5708*TAU*A*COSF(TAO)**2*(1.-SINF(BETA))/(1.+A)+4.*
     1SINF(TAO)**2*COSF(TAO)**3*COSF(BETA)/3.+(CLVOL1)/2.
      CB=1.5708*TAU*A*COSF(TAB)**2*(1.-SINF(BETA))/(1.+A)+4.*
     1SINF(TAB)**2*COSF(TAB)**3*COSF(BETA)/3.+(CLVOL2)/2.
      DADTAU=1000.*W/(RHO*V**2)*(1./CB-1./CO)
      DIFB1=(CLSA2-CLSA1)/.002+CL*DADTAU
C
C     DIFFERENTIATION OF LIFT COEFF X AREA WITH RESPECT TO Z
C
      DELZ=0.001
      DELA=-DELZ*BEAM/SINF(TAU)/WETL**2
      DELS=BEAM*DELZ/(SINF(TAU)*COSF(BETA))
```

```
      DELL=DELZ/SINF(TAU)
      CLVOL3=(((WC-DELL)**2)*SINF(TAU)/BEAM+(2.*WC+WK-3.*DELL)
     1*SINF(BETA)/(COSF(BETA)*3.))/(2.*(WETL-DELL)*(CV**2))
    7 CLB3=0.5*CLVOL3
      CLSA3=(1.5708*(A-DELA)*TAU*(COSF(TAU)**2)*(1.-SINF(BETA))
     1/(1.+A-DELA)+4.*(SINF(TAU)**2)*(COSF(TAU)**3)*COSF(BETA)/3.
     2+CLB3)*(S-DELS)
      CLVOL4=(((WC+DELL)**2)*SINF(TAU)/BEAM+(2.*WC+WK+3.*DELL)
     1*SINF(BETA)/(COSF(BETA)*3.))/(2.*(WETL+DELL)*(CV**2))
    9 CLB4=0.5*CLVOL4
      CLSA4=(1.5708*(A+DELA)*TAU*(COSF(TAU)**2)*(1.-SINF(BETA))
     1/(1.+A+DELA)+4.*(SINF(TAU)**2)*(COSF(TAU)**3)*COSF(BETA)/3.
     2+CLB4)*(S+DELS)
      DIFC1=(CLSA4-CLSA3)/0.002
C
C     DIFFERENTIATION OF MOMENT WITH RESPECT TO Z
C
      CPL1=.75-(WETL-DELL)/(5.21*(CV*BEAM)**2/(WETL-DELL)**2+2.39)
      CPL2=.75-(WETL+DELL)/(5.21*(CV*BEAM)**2/(WETL+DELL)**2+2.39)
      DCPLDZ=(CPL2-CPL1)/.002
      C1=0.5*RHO*(V**2)*DIFC1
      DMDZ=(CG-CPL)*C1/COSF(TAU)-W*DCPLDZ/COSF(TAU)
      A1=W/32.2+VERM
      B1=0.5*RHO*V*DIFB1
      C1=0.5*RHO*(V**2)*DIFC1
      D1=VERM*VCG*(COSF(TAU)*(CG-.5*WETL)/VCG-SINF(TAU))
      E1=B1*(COSF(TAU)*(CG-WETL/2.)-VCG*SINF(TAU))
      G1=V*B1
```

```
      A2=YI+VERYI
      B2=E1*((CG-CPL)*COSF(TAU)-VCG*SINF(TAU))
      C2=G1*((CG-CPL)*COSF(TAU)-VCG*SINF(TAU))+W*(CPL*(SINF(TAU)-VCG*
     1COSF(TAU)/SINF(TAU)/WK*(COSF(TAU)/SINF(TAU)-1./SINF(TAU)))-CG*
     2SINF(TAU)-VCG*COSF(TAU))
      D2=VERM*(COSF(TAU)*(CG-WETL/2.)-VCG*SINF(TAU))
      E2=B1*((CG-CPL)*COSF(TAU)-VCG*SINF(TAU))
      G2 = DMDZ
      E1=E1+V*VERM
      G1=G1+V*B1
      B2=B2+V*D2
      C2=C2+V*E2
C
C     U IS AN ARBITRARY NONDIMENSIONALIZING VELOCITY
      U = 10.
C
C     QUANTITY 0.5 RHO CANCELLED OUT OF ALL FOLLOWING
C
      A11=A1/(BEAM**3)
      B11=B1/(U*(BEAM**2))
      C11=C1/(BEAM*(U**2))
      D11=D1/(BEAM**4)
      E11=E1/(U*(BEAM**3))
      G11=G1/((BEAM*U)**2)
      A22=A2/(BEAM**5)
      B22=B2/(U*(BEAM**4))
      C22=C2/((BEAM**3)*(U**2))
      D22=D2/(BEAM**4)
```

```
      E22=E2/(U*(BEAM**3))
      G22=G2/((BEAM*U)**2)
      AA=1.
      BB=(A22*B11+A11*B22-D22*E11-E22*D11)/(A11*A22-D11*D22)
      CC=(A22*C11+B22*B11+A11*C22-D22*G11-E22*E11-G22*D11)/(A11
     1*A22-D11*D22)
      DD=(B22*C11+B11*C22-E22*G11-G22*E11)/(A11*A22-D11*D22)
      EE=(C22*C11-G22*G11)/(A11*A22-D11*D22)
      RETURN
      END
```

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