REGRESSION ANALYSIS
OF
SYSTEMATIC SERIES CALM WATER PERFORMANCE

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Abstract

The high-speed displacement vessels of round-bilge, transom-stern hull form have been widely used for various naval application. Systematic research leading to the better performance prediction of these vessels has been conducted at the Australian Maritime Engineering Cooperative Research Centre (AMECRC). This paper presents the analysis of the calm water experiments of the AMECRC systematic series of high-speed displacement mono-hull forms.

Speed independent regression analysis was performed and two independent sets of regression equations were derived from the same set of experimental data. The regression analysis was developed using a 'classical', multiple linear regression analysis, as well as a novel technique - nonlinear estimation. The nonlinear estimation approach proved to be superior and evaluation of the developed prediction methods has been presented

A unique data presentation method, particularly useful for systematic series data has been illustrated in this paper. It enables single-chart visualisation of multiparameter variations.

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Nomenclature

A_T/A_X	Ratio of area at transom and maximal sectional area
B	Beam on waterline
BM	Metacentric radius
B_T/B_X	Ratio of beam at transom and maximal beam
C_B	Block coefficient
C_F	Model-ship correlation line (ITTC 1957)
C_M	Midship section coefficient
C_P	Prismatic coefficient
C_R	Coefficient of residuary resistance
C_S	Wetted surface area coefficient
C_{WP}	Waterplane area coefficient
Fn	Froude number based on length
Fn_∇	Froude number based on volume
L	Waterline length
LCB	Longitudinal centre of buoyancy
S	Wetted surface area
R_R	Residuary resistance
T	Draft
W	Model weight
WSA	Wetted surface area
∇	Model volume

INTRODUCTION

Round-Bilge Hull-Forms

As described in Bailey (1974), semi-displacement hulls have a characteristic underwater shape distinguished by curvature between the bottom and sides, known as round bilge or soft chine. They have straight entrance waterlines, straight buttock lines and a transom stern. Their application is in the workboat, launch, frigate and corvette field. The extremes of the range are the heavy displacement, low speed workboat and the light displacement, fast patrol boat. The same author suggests that for Fn below 1.05 this type of form consistently offers a better performance in calm water than hard chine vessels. More detailed recommendation regarding this comparison could be found in Blount (1995). However, in naval and many other applications, the well proven high-speed monohull vessels still dominate, as stated in Lahtiharju et al. (1991). Savitsky outlines that the major interest of the maritime community for round-bilge hull forms is exemplified by the impressive growth in the number of high-speed ferries, and special purpose marine vehicles which utilise these hull forms. The resistance per unit weight of these craft is significantly less than for planing hulls, and they have substantially larger useful load fractions.

Systematic Series

A systematic series of models is obtained when parameters of one or more parent hulls are varied in a systematic manner. Usually not more than four parameters are varied. Data of this kind provide the designer with very valuable information regarding the parameter variation influence on the vessel's performance. Systematic series data can be presented by design charts or, more commonly nowadays, in mathematical form using regression equations. A comprehensive literature review is presented by Lewis (1988).

Regression Analysis

In regression analysis of ship resistance data many different mathematical models have been used. A detailed historical review of regression analysis in resistance predictions could be found in Fung (1991) and Farlie-Clark (1975). They could be broadly categorised into the two groups: speed-independent and speed-dependent models. Fung et al. (1993) provided a detailed discussion of these two types of models with their associated strengths and weaknesses. Briefly, speed independent models do not include ship speed as an independent variable and a separate analysis has to be performed at each of the analysed speeds. In speed-dependent regression models, ship speed is explicitly included as an independent variable and the mathematical model reflects the natural variation of resistance with speed (i.e. humps and hollows).

For naval engineering applications, the multiple linear regression analysis is most commonly used. It tries to identify the subset of the given set of independent variables which provides the best cause-effect explanation among the analysed data. This is a very tedious process, even using computers, and automated procedures are available. The most common ones are 'backward elimination' and 'forward selection' procedures, while more complex variations also exist. These selection procedures are based on preset selection criteria, which should be appropriately set to match the purpose of the analysis. Detailed descriptions could be found in Fung (1991) and StatSoft™ (1994).

The multiple linear regression model is presented in Equation 1. By including some transformations of independent variables (i.e. $X_2=X_1^2$, $X_3=X_1^3$,...), the model known as 'nonlinear in variables' could be obtained. The regression equation coefficients are obtained by least squares minimisation. The statistical package used provided numerous methods for data analysis. Apart from the multiple regression approach, another suitable method was identified: nonlinear estimation. This method actually presents the generalisation of the multiple regression analysis. It enables any relationships between the dependent and independent variables to be explored, as in Equation 2, unlike linear ones as in Equation 1. It is also possible to use any form of loss function. In other words, if a linear model (Equation 1) and least square loss function are used in nonlinear estimation, the results will be the same as in the multiple regression analysis. The comparison between multiple regression analysis and nonlinear estimation is discussed further in the paper.

$$Y = a + b_1 \cdot X_1 + b_2 \cdot X_2 + \ldots + b_p \cdot X_p \qquad (1)$$

$$Y = F(x_1, x_2, \ldots, x_n) \qquad (2)$$

AMECRC SYSTEMATIC SERIES

Background

The AMECRC systematic series is based on the High Speed Displacement Hull Form systematic series (HSDHF) developed at the Maritime Research Institute Netherlands (MARIN). The HSDHF project was a major research project on combatant vessels design, jointly sponsored by the Royal Netherlands Navy, the United States Navy, the Royal Australian Navy and MARIN. It was initiated by the growing belief that a significant improvement in the performance of transom stern, round-bilge monohulls could be obtained, especially with regard to their seakeeping characteristics.

The various aspects of HSDHF series development and extensive testing in calm water and waves are described in Blok, et al. (1984), Van Oossanen, et al. (1985), Koops (1985), Robson (1987), MARIN Reports (1982, 1987, 1989). Briefly, the series parent hull was selected because of its superior motion characteristics in waves, and the series was obtained by systematic variations of L/B, B/T and C_B parameters over the parameter ranges described in Table 1. About forty models were tested. The hull geometry and the test results have not been published to date, except for the parent design.

	HSDHF	AMECRC
L/B	4 - 12	4 - 8
B/T	2.5 - 5.5	2.5 - 4.0
C_B	0.35 - 0.55	0.396 - 0.50

Table 1 - Ranges of varied parameters for MARIN and AMECRC systematic series.

Series Establishment

In one of the many industry driven projects the initiative was taken to perform research similar to the MARIN HSDHF project. Industry involvement included all major Australian naval vessel shipbuilding companies, who identified the range of parameters (parameter space) relevant to their needs, as outlined in Table 1. The AMECRC series parent hull is identical to HSDHF series parent hull; the body plan is presented in Figure 1. The parameters which were kept constant

throughout the series are presented in Table 2. The AMECRC series used a different C_B transformation procedure than in the HSDHF series.

Title:
Creator:
CreationDate:
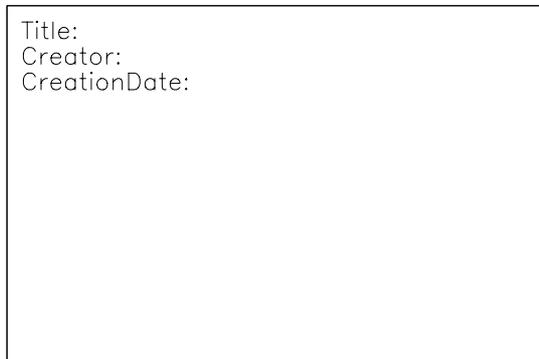
Figure 1 - Parent hull of AMECRC systematic series

C_P	0.626
C_{WP}	0.796
A_T/A_x	0.296
B_T/B_x	0.964
C_M	C_P x C_B
LCB	44.6% L_{PP} from transom

Table 2 - Hull parameters common for all series' models

A hull with $C_B = 0.55$ was derived from the parent hull ($C_B = 0.40$) and a software was developed which is capable of generating any hull with intermediate C_B value.

Testing Procedure

The models were tested in the Ship Hydrodynamics Centre at the Australian Maritime College. All models were constructed with a waterline length of 1.6 m. Calm water tests were conducted at speeds from 0.4 to 4 m/s, corresponding to Fn from 0.1 to 1.0. During testing, the models were free to sink and trim, and resistance, trim and rise of centre of gravity were recorded.

DATA PRESENTATION

Data Plotting

Any data analysis, at some stage, involves data plotting. With the computer power now available, data plotting is more flexible and easier than ever. It is quite straight-forward when the analysed data is based on one or two independent variables. However, with more independent variables, data presentation becomes more complex, as data must be presented as a set of charts. The number of data charts significantly increases with an increase in the number of independent variables, especially if the influence of each of independent variables is to be clearly presented.

The NPL series' model test data presentation of Marwood et al. (1969) seems to be very comprehensive. However, its deficiency is revealed as soon as the effect of L/B variation is analysed. In order to evaluate effect of L/B on resistance, serious mental effort is to be made to extract this information from existing plots.
In the case of the AMECRC systematic series, the effects of four independent variables (L/B, B/T, C_B and nondimensional speed) were analysed. Trying to cover every aspect of their variation would result in an impractical number of charts. Trying to clearly present the influence of every parameter may result in too many plots with not much data on each. This data fragmentation may result in failure to extract all of the available information.Therefore, a way was sought to reduce the number of charts, while preserving the amount of information.

Multiparameter Data Presentation

The cube on top of Figure 2 presents the series parameter space and is free to rotate around any of its axes. Different line types, used for its edges, represent variation of different independent parameters. The cube is first projected onto the 'horizontal' plane. This projection is then projected onto the horizontal axis of the intended plot. In this way, a function of L/B, B/T and C_B

(and the angles of rotations) is obtained, against which the R_R/W values are plotted. When the R_R/W values are connected by line types corresponding to the ones used in the parameter cube, the influence of the parameters' variation is clearly illustrated and a plot as in Figure 3 is obtained.

It is suggested to mark one of the models on the plot, the parent hull or the model with the
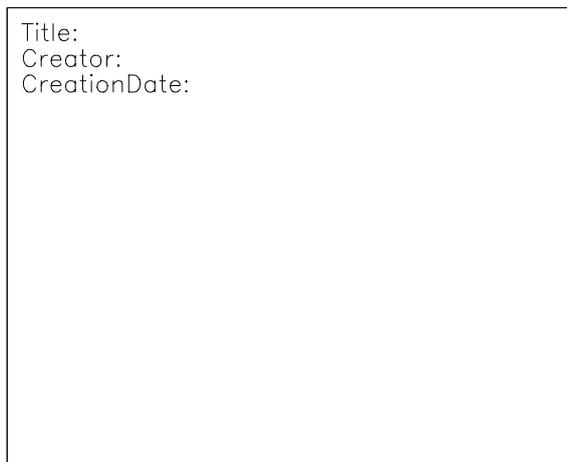
Figure 2 - Creation of four-dimensional R_R/W (L/B, B/T, C_B) plot for particular Fn_∇
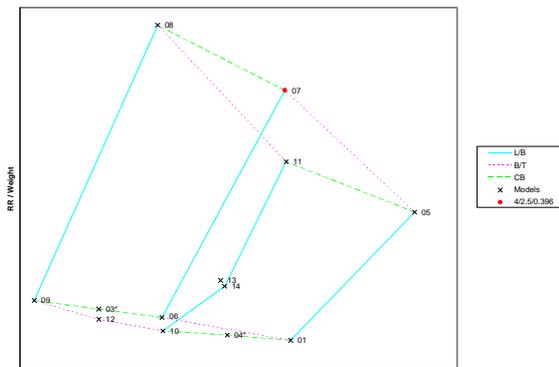
Figure 3 - R_R/W plot for $Fn_\nabla = 1.05$

lowest/highest values of all independent parameters, in order to enable easy determination of the direction of parameter changes. For example, in Figure 3 the model with the lowest parameters (L/B = 4, B/T =2.5, C_B=0.395) is marked and each line that starts from it represents an increase of that parameter. Therefore, it could be seen from this Figure that, at this particular Fn_∇, the increase of L/B, B/T and C_B results in

respective decrease, decrease and increase of R_R/W. Also, it could be seen that the effect of changes in B/T and C_B is less pronounced for higher L/B values.

In the analysis of the AMECRC series' data, the multiparameter charts lead to better understanding of the influences and interactions of the analysed parameters. Initially it was expected that these charts could be used as an ultimate performance prediction tool. The consequent data regression analysis revealed nonlinear effects which could not be accurately predicted from the chart. For example, Figure 4 presents the regression model fitted to the data from Figure 3. However, the multiparameter charts provided comprehensive data visualisation which supported decision making in various stages of the regression analysis.
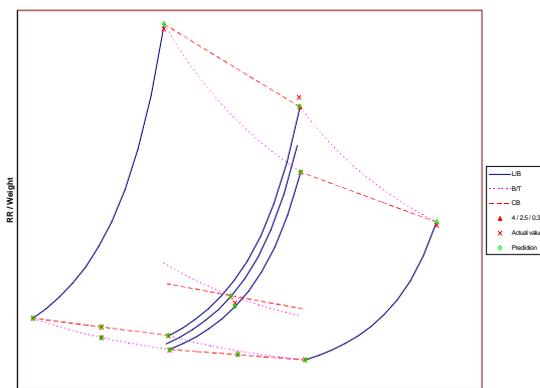
Figure 4 - Regression model for R_R/W at $Fn_\nabla = 1.05$

REGRESSION ANALYSIS

Development

A speed independent regression model was used in the analysis of the AMECRC series calm water resistance data. This means that a separate data analysis was performed at each of the nondimensional speeds.

In the literature, speed-independent regression models are either of C_R-Fn or R_R/W- Fn_∇ form.

The relationship between these formats is presented by Equations 3 and 4.

$$C_R = \frac{R_R}{\frac{1}{2} \cdot r \cdot S \cdot V^2} = \frac{R_R}{W} \cdot \frac{\nabla^{2/3}}{\frac{1}{2} \cdot S \cdot Fn_\nabla^2} \qquad (3)$$

$$Fn = \frac{Fn_\nabla}{\sqrt{\frac{L}{\nabla^{1/3}}}} \qquad (4)$$

Although a clear correlation between nondimensional parameters exists, the combinations of these parameters are completely independent. This means that the performance of any model at any speed will be linked to two independent data sets, on the basis of the nondimensional speeds. The consequent analyses will lead to independent conclusions. The importance of this approach is in the fact that more than one set of independent conclusions could be derived from the same set of data.

$$\frac{L}{\nabla^{1/3}} = \sqrt[3]{\frac{\left(\frac{L}{B}\right)^2 \cdot \frac{B}{T}}{C_B}} \qquad (5)$$

The encouraging outcome of this analysis had the following form:

$$R_R / Weight = b_1 + b_2 \cdot L / \nabla^{1/3} + b_3 \cdot \frac{1}{L / \nabla^{1/3}} \quad (6)$$

This form of regression equation agreed well with the literature. But, still this form was suggesting that the performance of the analysed vessels is proportional both to $L/\nabla^{1/3}$ and to its reciprocal value. That left the feeling that more specific conclusions could be extracted from the analysed data. It was decided to conduct more aggressive data analysis, as it was believed that the regression model from Equation 6 actually was suggesting that the best predictor has the following form:

$$\left(\frac{L}{B}\right)^{m_1} \cdot \left(\frac{B}{T}\right)^{m_2} \cdot C_B^{m_3} \qquad (7)$$

The final set of independent variables consisted of 86 variables of the form presented in Equation 7, where:

$$m_1 = \frac{0}{3}, -\frac{2}{3}, -\frac{4}{3}, -\frac{6}{3}$$
$$m_2 = \frac{0}{3}, -\frac{1}{3}, -\frac{2}{3}, -\frac{3}{3} \qquad (8)$$
$$m_3 = \frac{0}{3}, +\frac{2}{3}, +\frac{4}{3}, +\frac{6}{3}$$

These variables were used in the 'classical' approach, using multiple regression analysis (MRA). At the same time, two additional mathematical models were developed using nonlinear estimation technique. They attempted to fit the following mathematical model to the analysed data:

$$b_1 + b_2 \cdot \left(\frac{L}{B}\right)^{b_3} \cdot \left(\frac{B}{T}\right)^{b_4} \cdot C_B^{b_5} \qquad (9)$$

by minimising two different loss functions, described in Equations 10 and 11. Equation 10 presents the least squares function which is also 'built into' the multiple regression analysis model. Minimising of this function minimises absolute prediction errors, but results in larger relative errors for the models with lower values of resistance coefficients. Therefore the loss function, which minimises the relative prediction error, as in Equation 11, was used. These two models are marked as NLE1 and NLE2 in Figures 5 and 6.

$$(\text{observed - predicted})^2 \qquad (10)$$

$$\left(\frac{\text{observed - predicted}}{\text{observed}}\right)^2 \qquad (11)$$

The comparison of six developed mathematical models was performed on the basis of R^2 value based on the relative prediction differences and is presented in Figures 5 and 6. Based on this, it was concluded that the nonlinear estimation model, with the loss function based on the relative errors, provides the best estimation. Also, it could be seen that both formats (C_R-Fn and R_R/W- Fn_∇) demonstrate a similar level of accuracy.
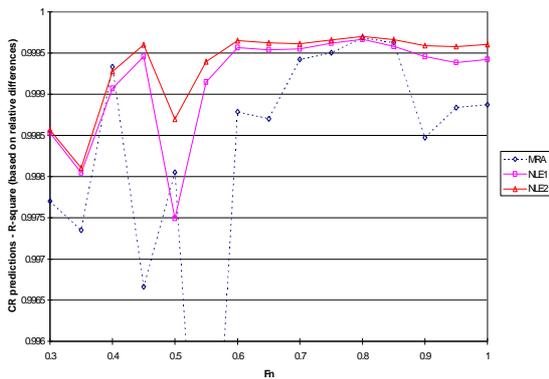
Figure 5 - C_R-Fn predictions - relative R^2
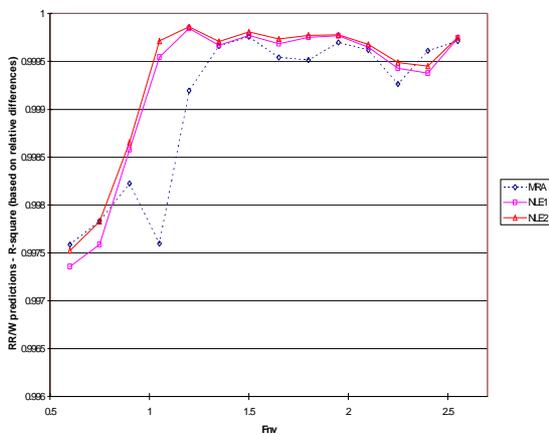
Figure 6 - R_R/W- Fn_∇ predictions- relative R^2

Figure 7 - Predictions of model #13 performance - C_R-Fn plot

Evaluation

The best way to evaluate a regression equation is to use it to predict the performance of a model not used in its development. The two most accurate regression equations, as identified above, were regenerated without model #13 data. This model has parameters in the middle of the series' parameter space. The newly obtained performance prediction equations were then compared with the test results of the model #13. The comparison are presented in Figures 7 and 8.

They have demonstrated the accuracy of about ±2% for prediction of residuary resistance, which implies higher accuracy for the total resistance prediction. The regression equations generated with and without model #13 were compared to explore the sensitivity of the mathematical models to the database selection. The differences were within 1.5%.
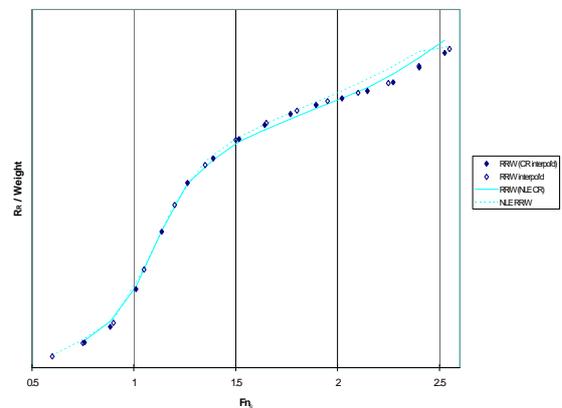
Figure 8 - Predictions of model #13 performance - R_R/W- Fn_∇ plot

Software implementation

Two sets of regression equations were developed, as described above. The C_R-Fn set covers a range of Fn from 0.3 to 1.0, and R_R/W- Fn_∇ a range of Fn_∇ from 0.6 to 2.55. They were implemented as an easy-to-use spreadsheet application.

In order to obtain maximum benefits from the model test data, a prediction correction was implemented in the software application. It is based on the difference between performance prediction and the test results for the series' model with the closest length-volume ratio.

Discussion of Results

As mentioned earlier the length-volume ratio is the best descriptor of the performance of round-bilge monohull forms. That was confirmed by the mathematical model obtained from the initial regression analysis, presented in Equation 6. In that mathematical model the influence of series parameters on the calm water resistance was actually described by their influence on the length-volume ratio, as per Equation 5. Further data analysis showed that even more subtle effects of series' parameters on vessel's performance could be identified. This enabled the evaluation of the series' parameters influence even for the vessels with the same value of length-volume ratio. In order to predict the total resistance it is necessary to obtain an estimate of frictional resistance. It is calculated using the following formula:

$$R_F = C_F \cdot \tfrac{1}{2} \cdot r \cdot S \cdot V^2 \qquad (12)$$

where C_F is the model-ship correlation line as per ITTC 1957 formula, described by Lewis (1988). It is obvious that an accurate estimation of the wetted surface area is needed in order to evaluate this formula. Dynamic wetted surface area was not recorded during the tests, but regression analysis of the wetted surface area at rest was performed. In order to increase the prediction accuracy, additional models were computer generated, so that the hull database for this analysis consisted of 27 models. The influence of the series' basic parameters on the wetted surface area coefficient could be seen in Figure 9.

CONCLUDING REMARKS

This paper describes the work done so far in the area of the calm water resistance of the AMECRC systematic series of round-bilge monohulls. Based
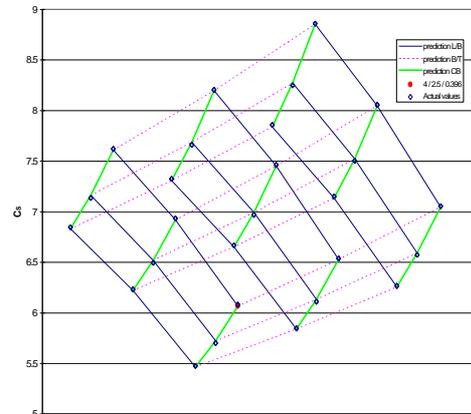
Figure 9 - NLE prediction of C_S - prediction vs actual values

on the same set of test values, two independent sets of regression equations were developed, in C_R-Fn and R_R/W- Fn_∇ formats. Each set was generated using two different regression techniques - 'classical', multiple regression analysis and a novel, nonlinear estimation method. The later proved to be superior.

Also, a data presentation method is presented which enables single chart visualisation of multiparameter changes. This method is particularly useful for presentation of systematic series data.

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