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WAVE RESISTANCE OF WAVE_ PIERCING CATAMARANS

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ABSTRACT: Present paper researches the theoretical method for calculating the wave resistance of wave-piercing catamaran. As an example the wave resistance of a wave-piercing catamaran is calculated. The comparison among the theoretical calculation results, the model test results and the graphic evaluation results shows the efficiency of present theoretical method. Finally the wave resistance regulation of wave-piercing catamarans is investigated based on the theoretical method.

KEY WORDS: wave-piercing catamaran, wave resistance, ship form, high performance vehicle

1. INTRODUCTION

In 1983 the new concept of wave-piercing catamaran (WPC) was proposed by an Australian scholar who combined the virtues of the hull forms of SWATH and deep V ships, the structure style of ordinary catamarans and the arc stud of hydrofoils, and overcame the shortcomings of each. The WPC has fine stability, high speed and good sea-keeping performance. The navigation area applied to the WPC is rather broad. Moreover it is convenient to general layout with the spacious deck. With their fine performance in total, the WPC gets a wide application scope both in military and in commerce. After its successful development at the end of 80′s, the WPC were quickly put into production. Now a large number of WPCs operate in the ocean area of Australia, Europe and Asia.

It is known that the WPC is a kind of special high-speed catamarans, the geometry of which is different from that of the ordinary ships in the following respects. (1) With the long chine line, the underwater part of its each piece hull is similar to deep V ship form; (2) The piece hull is slender, the slenderness Ψ of which is larger than 7.5～8.5, even larger than 9.0, as reported in some papers, L/B of which is large than 12～14, the enchant angle of which is less than 7～11 degree, even 6 degree; (3) The piece hull has the transom. The theoretical method of wave resistance calculation applied to the WPC has not yet reported. The present paper researches the theoretical method for calculating the wave resistance of wave-piercing catamaran, and the influence to the wave resistance of the piece hull slenderness Ψ, the space 2b between two piece hulls is investigated based on the method. the above-mentioned work is significant to the hull form optimization and the speed prediction of WPC.

The famous Michell integral can be inferred to the linear wave resistance formula of catamarans[1], which could be called the Michell-type wave resistance formula of catamarans. This sort of calculation method requires less amount of calculation, and to some ship form, one can get rather accurate results in wide speed range. However the influence of chine line and transom on wave resistance are not involved in the Michell-type wave resistance formula of catamarans, which must be taken into account in the wave resistance calculation of WPC.

Another method of wave resistance calculation is to solve the velocity potential according to linear or nonlinear free surface conditions and exact body surface condition by source distribution and evaluate wave resistance based on the velocity potential. The distributed source could be the Rankine source or the Havelock source, which was used in the Dawson method and the method presented in Ref. [2]. Generally speaking, the above methods could be called surface element methods, the accuracy of which is increased more or less but the calculation amount of which is increased significantly. They have been already used in wave resistance calculation of regular monohull ships, but are rarely used in wave resistance calculation of catamarans let alone WPCs.

In the present paper, the Michell-type wave resistance formula, is incorporated with both equivalent water line method and transom sink point method to include the influence of transom and the chine line influence is taken into account through increasing water line number and improving "tent function"[3] of expressing ship form. The wave resistance of a WPC model[4] is calculated with the above-mentioned method. The results show that the theoretical value of the wave resistance is rather close to the measured data, and has the same trend with Keuning's 25 ° deadrise angle graphic results in the high-speed range[5]. Finally a WPC series is developed by systematically changing the space 2b between two piece hulls and the piece hull slenderness Ψ with the above WPC as parent model and the theoretical wave resistance graph reflecting the influence of 2b and Ψ is given based on the corresponding theoretical method.

2 CALCULATION METHOD OF WPC WAVE RESISTANCE

2.1 *Michell-type calculation method of catamaran wave resistance*

The coordinate axies are taken as in Fig. 1, and through deduction[5] the Michell-type formula of wave resistance can be expressed as

$$R_W = 8\pi\rho K_0^2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2[1 + \cos(2bK\sin\theta)][P^2(\theta) + Q^2(\theta)]\sec^3\theta d\theta \tag{1}$$

$$\begin{Bmatrix} P(\theta) \\ Q(\theta) \end{Bmatrix} = \frac{U}{2\pi} \iint_{S_0} \frac{\partial f(x,z)}{\partial x} \begin{Bmatrix} \cos \\ \sin \end{Bmatrix} (K_0 x \sec\theta) \exp(K_0 z \sec^2\theta) dx dz \tag{2}$$

where

$K_0 = \frac{g}{U^2}$

$K = K_0 \sec^2\theta$

R_W wave resistance

ρ	water density
g	gravitational acceleration
U	ship speed
$2b$	space of the longitudinal center lines of piece hulls
x, z	coordinates of the distributed source and sink
f	ship hull function
S_0	region of the longitudinal center plane

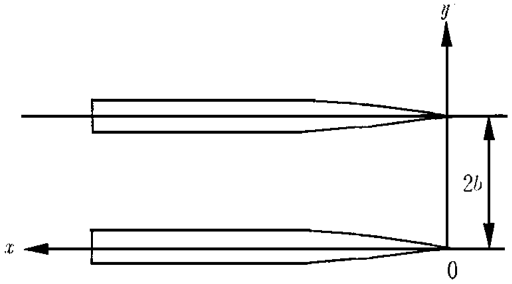


Fig. 1 Coordinate illustration

2.2 *Amendment method for transom influence*

According to the Lagally theory[6], the summation of potential pressure on body surface equals the one on any flow surface surrounding the body surface. So the wave resistance of transom ships could be calculated by adding an appendage water line length after transom, which is called the "equivalent water line method". Corresponding to equivalent water line method, in wave resistance calculation the flow surface including appendage water line length should be regarded as the "equivalent body surface" and the wave resistance should be calculated from the equivalent body surface. This is one method adopted by the present paper and it requires the values of the appendage water line length, which is taken from the measurements of a round bilge boat model[7].

Another method coping with the transom influence adopted in the present paper is to arrange a point sink in the transom centroid, the strength of which equals the summation of the distribution source strength in the longitudinal center plane and is expressed as

$$\sigma_T = -\iint_{S_0} \sigma dS = -\frac{U}{2\pi}\iint_{S_0} \frac{\partial f}{\partial x} dx dz \approx -\frac{U}{2\pi}\left(\frac{A_T}{2}\right) \tag{3}$$

where σ_T is the strength of the point sink, and A_T is the projection area of the transom. The wave resistance formulae including the transom influence can be written as

$$R_W = 8\pi\rho K_0^2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2[1 + \cos(2bK\sin\theta)][P_t^2(\theta) + Q_t^2(\theta)]\sec^3\theta d\theta \tag{4}$$

$$\begin{Bmatrix} P_t(\theta) \\ Q_t(\theta) \end{Bmatrix} = \begin{Bmatrix} P + P_T \\ Q + Q_T \end{Bmatrix} = \begin{Bmatrix} P \\ Q \end{Bmatrix} + \sigma_T \begin{matrix} \cos \\ \sin \end{matrix}(K_0\sec^2\theta x_T) \cdot e^{-K_0\frac{z_T}{2}\sec^2\theta} d\theta \tag{5}$$

$$w_T = x\cos\theta + y\sin\theta = x\cos\theta = 2l\cos\theta \tag{6}$$

In the above formulae P_t and Q_t are the Kochin functions including the point sink contribution, and Z_T the immersion depth of the transom. Through the disposal of (3), (4), (5), Eq. (4), can be reduced to

$$R_W = R_{W1} + R_{W2} + R_{W3} \tag{7}$$

in which

$$R_{W1} = 8\pi\rho K_0^2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2[1 + \cos(2bK\sin\theta)](P^2 + Q^2)\sec^3\theta d\theta \tag{8}$$

$$R_{W2} = 8\pi\rho K_0^2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2[1 + \cos(2bK\sin\theta)]2(PP_T + QQ_T)\sec^3\theta d\theta \tag{9}$$

$$R_{W3} = 8\pi\rho K_0^2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2[1 + \cos(2bK\sin\theta)]\sigma_T^2[\exp(-K_0\sec^2\theta\frac{z_T}{2})]^2\sec\theta d\theta \tag{10}$$

$$\left\{\begin{matrix} P_T(\theta) \\ Q_T(\theta) \end{matrix}\right. = \sigma_T \begin{matrix} \cos(K_0\sec^2\theta w_T) \\ \sin(K_0\sec^2\theta w_T) \end{matrix} \exp(-K_0\sec^2\theta\frac{z_T}{2}) \tag{11}$$

with transom point sink method there is no need of calculating the values of appendage water line length. Although the above two methods are approximate, they would not introduce large error for the wave resistance difference caused by the transom is small compared with the total wave resistance.

2.3 *Numerical method and influence of chine line*

The above mentioned WPC has a long chine line. This form of hard chine ship is mainly used as the ship form of planning boats. The investigation of chine line influence in the wave resistance calculation is not much, for the wave resistance of planing boat in planing navigation state could be almost ignored. In the present paper the chine line influence will be taken into account in numerical analysis.

Firstly, the ordinary method of tent function is introduced for calculating the Michell integral, where the chine line influence is taken into account by regularly increasing the water line numbers. By this method, the water lines respectively pass through each intersection point of the station lines and the chine line, so that the offsets for wave resistance calculation fully reflect the chine line influence. In spite of simplicity, the water line numbers and calculation amount required increase greatly.

Secondly, an improved method of tent function is proposed to treat the chine line influence, by which the chine line influence is taken into account without increasing the water line numbers. The improved unit tent function is defined as follow

$$F^{(i,j)}(x,z)=\begin{cases}\left(1-\dfrac{x_i-x}{x_i-x_{i-1}}\right)\bullet\left(1-\dfrac{z_{ji}-z}{z_{ji}-z_{j-1,i}}\right) & x_{i-1}\le x\le x_i,\ z_{j-1,i}\le z\le z_{ji}\\ \left(1-\dfrac{x_i-x}{x_i-x_{i-1}}\right)\bullet\left(1-\dfrac{z_{ji}-z}{z_{j,i}-z_{j+1,i}}\right) & x_{i-1}\le x\le x_i,\ z_{ji}\le z\le z_{j+1,i}\\ \left(1-\dfrac{x_i-x}{x_i-x_{i+1}}\right)\bullet\left(1-\dfrac{z_{ji}-z}{z_{ji}-z_{j-1,i}}\right) & x_i\le x\le x_{i+1},\ z_{j-1,i}\le z\le z_{ji}\\ \left(1-\dfrac{x_i-x}{x_i-x_{i+1}}\right)\bullet\left(1-\dfrac{z_{ji}-z}{z_{j,i}-z_{j+1,i}}\right) & x_i\le x\le x_{i+1},\ z_{ji}\le z\le z_{ji}\end{cases}\tag{12}$$

The definition domain of $F^{(i,j)}(x,z)$ is shown in Fig. 2. The vertical grid lines of the domain are station lines, but the transverse grid lines need not be water lines, even not be strainght lines, instead can be inclined straight lines or deflected lines, as shown in Figs. 2(a), 2(b) and 2(c), 2(d).

Like the ordinary unit tent function, $F^{(i,j)}(x,z)$ equals 1 at the central point of the domain and $F^{(i,j)}(x,z)$ equals 0 on the vertical border grid lines or at the border knot points of the domain. However $F^{(i,j)}(x,z)$ is not equal to 0 on the transverse border grid lines, unlike the ordinary unit tent function. From (12), the improved tent function approaching the curved surface of whole ship form can be expressed as

$$f^{(i,j)}(x,z)=\sum_{i=1,j=1}^{nx,nz}y_{i,j}\bullet F^{i,j}(x,z)\tag{13}$$

The improved tent function like (13) can approach the curved surface of ship form and depict the half breadth of the chine line, so that it can reflect the chine line influence on theory wave resistance.

3 EXAMPLES

3.1 *Model*

A WPC model (WPC901) is given in Ref. [4], which includes model data and model test results. The main parameters for the ship form are shown in Table 1.

Table 1 Main parameters of WPC901

Piece hull displacement ∇	0.0075m^3	Slenderness of piece hull Ψ	7.97
Total length L_t	1.74m	Length breadth ratio of piece hull L/B	11.47
Water line Length L	1.56m	Breadth draft ratio of piece hull B/T	2.194
Total breadth B_t	0.744m	Space length ratio $2b/L$	0.3077
Piece hull breadth B	0.136m	Deadrise angle of middle ship β	24°
Draft T	0.062m	Entrant angle of water line a	7°
Space between piece hulls $2b$	0.48m	Transom water line breadth B_T	0.136m
Wet surface area S	0.293m^2	Transom immersion depth Z_T	0.0266m

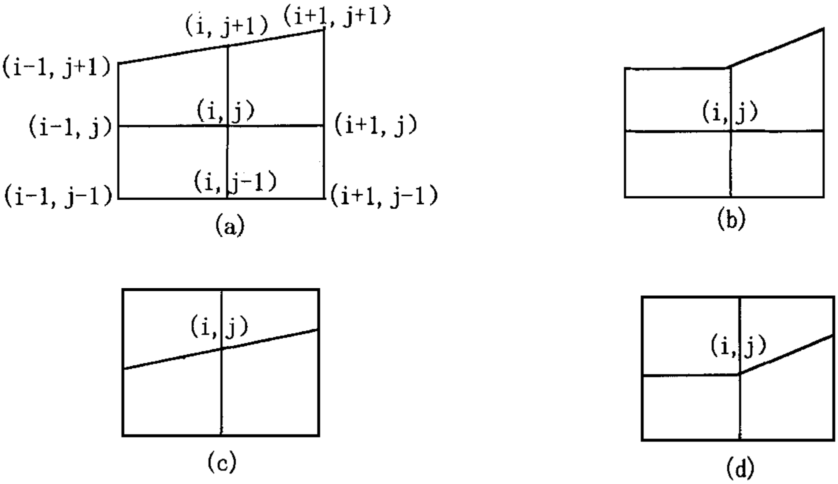


Fig. 2 Definition domain of improved tent function

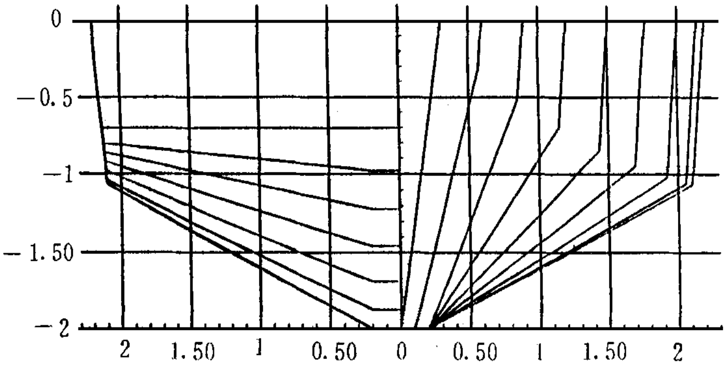


Fig. 3 Underwater body plan of WPC901

The underwater body plan of WPC901 is shown in Fig. 3. Researchers at Harbing Engineering University carried out model resistance test with this model at Dalian University of Science and Technology in November, 1993.

3.2 *Comparison among results from calculation, model tests and graphic evaluation*

As an example, the wave resistance of WPC901 is calculated by using the present method. Moreover, the calculation results of residual resistance are compared with those obtained in the model tests and the graphic evaluation to validate the accuracy of the theoretical method.

The WPCs are hard-chine ships, but operate in semi-planing state. There is not many graphs available for semi-planing hard-chine ships. The frequently used one is the series 62 graph[8]. However the mid-ship deadrise angle of the series 62 is 12.5 degree, which is much less than that of WPC. In Ref. [9], Keuning developed a new series by reforming series 62 from the mid-ship of 12.5 degree to the mid-ship of 25 degree. The other parameters of Keuning's series remain the same as those for the series 62 as much as possible. The ship form of Keuning's series is close to that of the WPC relatively so that the resistance of WPC could be evaluated more accurately based on the model test data of Keuning's than the series 62. To compare the accuracy of the theoretical method with that of the graphic method and to investigate the correlation of resistance between Keuning's series and the WPC, the residual resistance of WPC901 is tentatively evaluated by Keuning's series. The speed range of Keuning's graph includes the semi-planning speed and the planing speed ranges, but the greatest L/B of the graph is much less than that of the WPC, so that it requires extrapolation to evaluate the resistance with this graph and this is not convenient and reliable. By considering the semi-planing state of WPC, with the slenderness Ψ determined, in resistance calculation, the L/B influence on resistance is not taken into account tentatively, so that only the model test data of greatest L/B is used.

For convenience, the wave resistance curve obtained in the theoretical calculation, the model tests and the graphic evaluations are all plotted in Fig. 4 where the curve for theoretical wave resistance Cw is expressed by the theoretical residual resistance Cr and $Cr = 1.2\,Cw$. In the following, all the wave resistance curves are so treated in order to compare the residual resistances. It is shown in Fig. 4 that as $Fn = 0.35 \sim 1.0$, the theoretical curve of residual resistance exhibits the same regulation as the test residual resistance, and the error is small; as $Fn = 0.5 \sim 0.9$, the theoretical value is very close to the test value, and the accuracy is satisfactory to the engineering requirement. The difference between the graphic value and the test value is greater comparatively, yet as $Fn \geq 0.45$, the regulation of the graphic value and the test value is accordant. As to how the graph value is greater as $Fn \leq 0.45$, one explanation should be that the original wet surface area data of the series does not include the area of the sides in contact with the solid water[9]. This would introduce obvious error to decrease the evaluation value of the wet surface area and increase the evaluation value of the resistance.

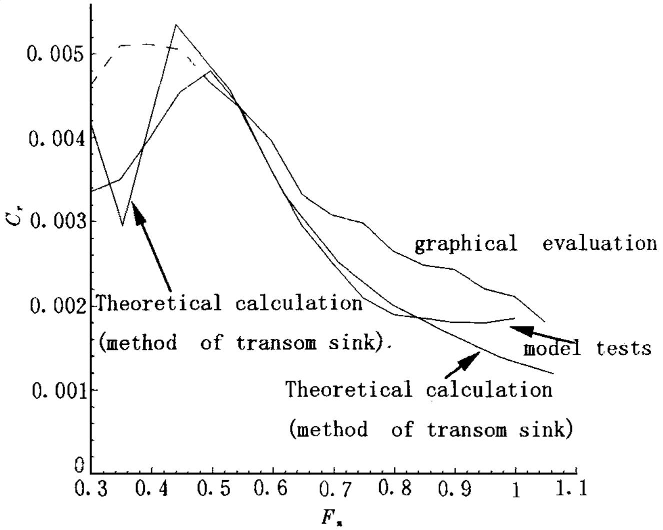
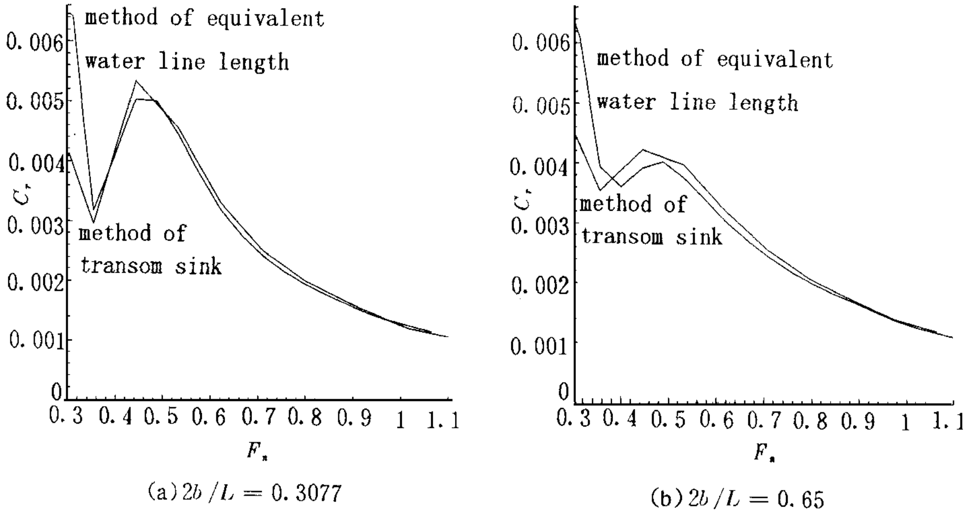


Fig. 4 Comparison among theoretical value, test value and graphic value of Cr of WPC901

According to the above calculation results, it could be concluded that the resistance evaluation graph of WPC can be developed based on the theoretical calculation method, the model test results, the existing graphs and the navigation data of full WPCs.

3.3 *Comparison among the different calculation results*



(a) $2b/L = 0.3077$ (b) $2b/L = 0.65$

Fig. 5 Comparison between two kinds of residual resistance of WPC901

Two methods for the treatment of transom influence and chine line influence are applied to the wave resistance calculation of WPC901. There is almost no difference between the results obtained with the two methods for the chine line treatment. Fig. 5 gives the Cr-Fn curves from the two methods of the transom effect consideration. The two kinds of the Cr-Fn curves coincide very well as $Fn > 0.35$; as $Fn < 0.35$, though the crest and the trough of the two curves all are exaggerated, the curve from transom sink point method is closer to the model test curve. The transom sink point method and the improved tent function method are convenient, for the former does not need the values of appendage lengths and the results are better in the low speed situation; and the later requires less computation amount. So it is desired that the combination of them could be developed to a regular calculation method for the wave resistance of WPC.

4 INFLUENCE OF THE SPACE 2*b* AND THE SLENDERESS Ψ

4.1 *The influence of the space 2b between the two piece hulls*

The space between the two piece hulls is one of the important parameters, and its proper selection should be favorable to decrease the wave resistance. To investigate the influence of $2b$ on wave resistance, the theoretical curves of Cr vs $2b$ and Fn is given with the present method through defining a group data of $2b$ as input data. The theoretical graph is shown in Fig. 6.

The influence regulation of $2b$ is clearly shown in the graph, in which a favorable interference region exists for the reduction of wave resistance in the vicinity of $Fn = 0.4$, and the unfavorable interference takes place between the two piece hulls to increase wave resistance in the vicinity of $Fn = 0.5$, but as $Fn > 0.75$, the favorable interference again takes place between the two piece hulls though the change of wave resistance is not apparent in this region.

In view of the importance of the main crest of the wave resistance curve of WPC and the increase of frictional resistance in the small space between piece hulls, it is proposed that without the consideration of the strength, price and technology, greater space should be selected to decrease wave interference. Moreover, as $2b > 0.6L$, the wave interference almost

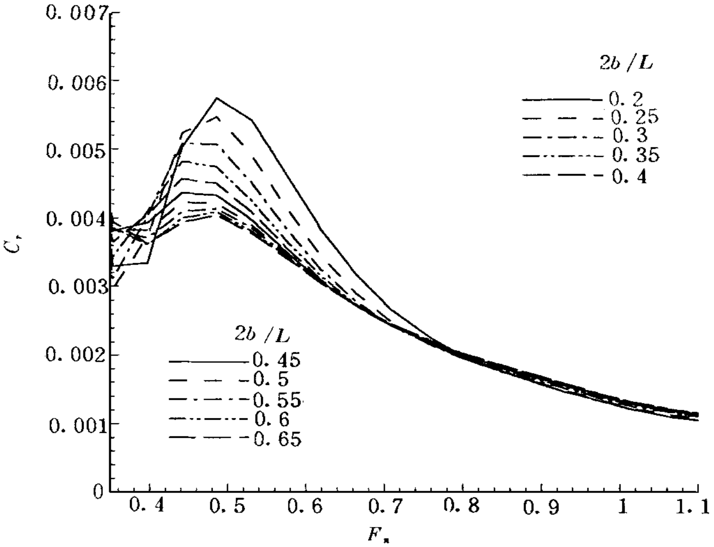


Fig 6 WPC graph about the influence of the space 2*b*

disappears

4 2 *Influence of the slenderness* Ψ

Figs 7 and 8 express the influence of the slenderness of piece hull Ψ is one of the most important parameters to influence the wave resistance and the total resistance Ψ of WPC exceeds the value of regular ships, so that it is not sonvenient to investigate the Ψ influence on resistance with ordinary graphs Developing the theoretical graph reflecting the Ψ influence on the resistance is of significance

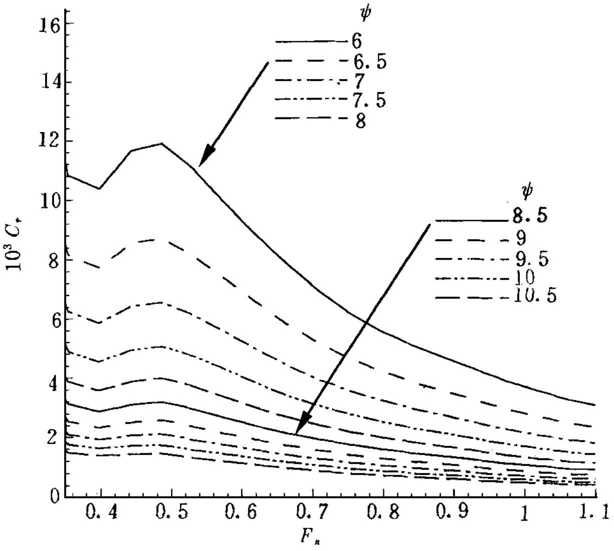


Fig 7 WPC graph about the influence of the piece hull slenderness Ψ

It is shown in Fig 7 that the wave resistance decreases with the increase of Ψ, however as Ψ reaches some values, the wave resistance curves become dense, which implies that

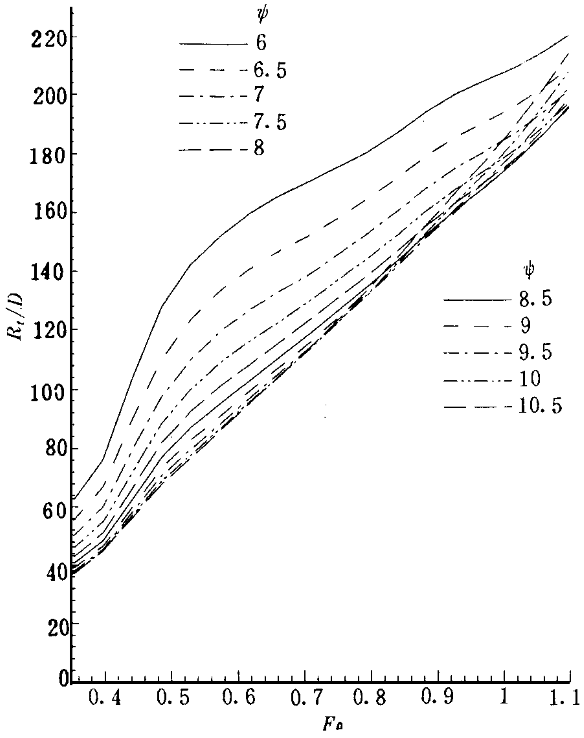


Fig. 8 Theoretical curve of R_t/D ~ Fn for piece hull

the Ψ influence to the wave resistance is not obvious. From Fig. 8, there are some intersection points on the total resistance curves, which implies the total resistance vs Ψ is not monotonous and exists the optimal Ψ corresponding to the minimum total resistance. These two regulations are the same as that of regular ship forms, which could guide the selection of Ψ in the design of WPC.

5. CONCLUSIONS

According the above analysis, it is concluded that

(1) The present theoretical method has high accuracy and can be used in practice;

(2) It is possible to give the resistance graph of WPC by incorporating of the theoretical calculation, the existing graphs, the model tests and the navigation data on full WPCs;

(3) The theoretical graph can correctly reflect the influence regulation of $2b$ and Ψ on resistance and can provide guidance to WPC design;

(4) Without the consideration of the strength, price and technology, it is favorable to select greater space between pierce hulls; and there exist some Ψ with little influence on the wave resistance and the optimal Ψ corresponding to the minimum total resistance.

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