The Relative Wave Elevation for the Trimaran Ship Advancing in Waves with Steady Flow Effect

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ABSTRACT:
The paper presents the three-dimensional source distribution method including the coupled effect of the steady flow potential and unsteady potential to predict the relative wave relative elevation for a trimaran ship advancing in regular waves. The potentials due to incident wave, diffraction wave, radiation wave and steady ship wave are considered to calculate the resultant wave elevation. The pulsating source and translating source are adopted to solve the corresponding unsteady potential and steady potential respectively. The steady potential affects not only trimaran ship motions but also the resultant wave elevation, consequently the relative wave elevation will be different from that without steady flow effect. The results calculated in the present study reveal that the combined effect on the unsteady motions and relative wave elevations due to the steady flow potential are indeed significant..

1 INTRODUCTION

As we know, the trimaran ship has superior seakeeping performance, which can be applied to both commercial and military purposes. Besides, it also has the following merits: (1) Superior fuel consumption efficiency due to its slender ship hulls. (2) Superior stability due to the suitable adjustment of the side hulls. (3) Low resistance at high speed due to its slenderness. Therefore it becomes a concerned vehicle with high performance recently.

The seakeeping problem for mono-hull ships in frequency domain have been studied by several authors using the strip theory, e.g., Kim et al. (1980). Based on the strip theory, Fang et al. (1993) further applied the time domain technique to analyze the nonlinear motions of a ship traveling in large waves. The three-dimensional theories for seakeeping analysis were also well developed for the mono-hull ship in waves, e.g. Inglis and Price (1981). The researches about twin-hull ships, either by strip theory or three-dimensional one, were also made such as Lee et al. (1973) and Lee (2000).

For the trimaran ship, the strip theory is not suitable because the significant interaction effect exists between the main hull and side hull. Some authors applied 2½ dimensional theory to analyze the related seakeeping problems such as Duan et al. (2001) who assumed the three-dimensional boundary condition and two-dimensional govern equation to predict the motions of multi-hulls. Begovic (2003) also used the same technique to discuss the structural problems of the trimaran ship. However, the three-dimensional theory such as Bingham et al. (2001) and Fang and Chen (2005) may still be the better one to treat the related problem for the trimaran ship. Up to now, most of the authors concentrated on the problems about motions and wave loads for the trimaran ship, however very few treat the relative wave elevation which is very important to analyze the wave impact on the connected deck between the min hull and side hull. In addition, the steady flow effect on the unsteady ship motions is usually neglected in the previous study, e.g. Kim et al. (1980) and Fang et al. (1993), however, it might play an important role in some cases as shown in Fang (2000) and Fang et al.(2007). Therefore the present study combined the steady flow effect with unsteady waves to calculate the relative wave elevation for the trimaran ships in waves using the three-dimensional theory. The related mathematical formulas are described in the following sections.

2 MATHEMATICAL MODEL

Assume a trimaran ship travels with constant speed U in regular waves and the incident wave amplitude and ship motion are very small, the resultant potential due to the steady and unsteady motions can be expressed as

$$\Phi\left(x,y,z,t\right)=\left(\phi_S\left(x,y,z\right)-Ux\right)+\mathrm{Re}\{[\phi_I\left(x,y,z\right)+\phi_D\left(x,y,z\right)+\sum_{j=1}^{6}\zeta_j\phi_j\left(x,y,z\right)]e^{-i\omega t}\} \tag{1}$$

where –Ux is the uniform flow, $\phi_s(x,y,z)$ is the steady flow potential, $\phi_I(x,y,z)$ is the incident wave potential, $\phi_D(x, y, z)$ is the diffraction potential, $\phi_j(x,y,z)$ is the radiation potential, and ζ_i is the motion displacement, $j=1,2,3,4,5,6$ represents surge sway, heave, roll, pitch and yaw, respectively. Using the three-dimensional source distribution method and boundary conditions, we solve the corresponding potentials by the corresponding boundary conditions. Then the exciting force, added mass, and damping coefficients can be obtained and equations of motions can be written as below,

$$\sum_{j=1}^{6}\left[-\omega^2\left(M+A_{ij}\right)-i\omega B_{ij}+C_{ij}\right]\zeta_j=F_i \qquad i=1,2,\ldots 6 \tag{2}$$

In equation (2), the suffix $i,j=1,2,3,4,5,6$ represent surge, sway, heave, roll, pitch, and yaw modes, respectively. M is the mass of ship, A_{ij} is the generalized added mass, B_{ij} is the generalized damping coefficient, C_{ij} is the ship hydrostatic restoring force, and F_i is the wave exciting force. The steady flow effect has been included in these hydrodynamic forces(Fang and Lin,2000)

Assume

$$W\left(x,y,z\right)=\nabla\left[\phi_S\left(x,y,z\right)-Ux\right] \tag{3}$$

and neglect the hydrostatic pressure, then the linearlized pressure combining the steady flow effect can be written as

$$P\left(x,y,z,t\right)=P_s(x,y,z)+[P_I(x,y,z)+P_D(x,y,z)+P_R(x,y,z)]e^{-i\omega t} \tag{4}$$

The pressure in equation (4) consists of the following four components:

Steady pressure

$$P_S\left(x,y,z\right)=-\frac{1}{2}\rho\left(W\left(x,y,z\right)^2-U^2\right) \tag{5}$$

Incident wave pressure

$$P_I\left(x,y,z\right)e^{-i\omega t}=\left(i\omega\rho\phi_I\left(x,y,z\right)-\rho W\left(x,y,z\right)\cdot\nabla\phi_I\left(x,y,z\right)\right)e^{-i\omega t} \tag{6}$$

Diffraction wave pressure

$$P_D\left(x,y,z\right)e^{-i\omega t}=\left(i\omega\rho\phi_D\left(x,y,z\right)-\rho W\left(x,y,z\right)\cdot\nabla\phi_D\left(x,y,z\right)\right)e^{-i\omega t} \tag{7}$$

Radiation wave pressure

$$P_R\left(x,y,z\right)e^{-i\omega t}=\left(i\omega\rho\sum_{j=1}^{6}\zeta_j\phi_j\left(x,y,z\right)-\rho W\left(x,y,z\right)\cdot\nabla\sum_{j=1}^{6}\zeta_j\phi_j\left(x,y,z\right)\right)e^{-i\omega t} \tag{8}$$

From the Bernoulli equation and the equation (4), the resultant wave elevation on the free surface is

$$\eta(x, y, 0, t) = \frac{1}{\rho g}\left\{P_S(x, y, 0) + \mathrm{Re}\left[\left(P_I(x, y, 0) + P_D(x, y, 0) + P_R(x, y, 0)\right)e^{-i\omega t}\right]\right\} \quad (9)$$

For simplicity, the equation (9) is replaced by

$$\eta(x, y, 0, t) = \eta_S(x, y, 0) + \mathrm{Re}\left(\eta_t(x, y, 0)e^{-i\omega t}\right) \quad (10)$$

where η_s is the wave elevation due to steady ship motion and η_t is the resultant wave amplitude due to the unsteady motion.

The vertical motion amplitude at any point on board can be written as (Kim et. al, 1980)

$$S_3(x, y, z) = \zeta_3 - x \cdot \zeta_5 + \frac{iU}{\omega}\zeta_5 + y \cdot \zeta_4 \qquad j = 1,2 \ldots 6 \quad (11)$$

From equations (10) and (11), we can derive the relative wave elevation as

$$\eta_r(x, y, 0, t) = \eta(x, y, 0, t) - \mathrm{Re}(S_3 e^{-i\omega t}) \quad (12)$$

The maximum and minimum relative wave elevations can be obtained by adding or subtracting the amplitude of the dynamic swell up from the steady wave, i.e.

$$\eta_{r\pm} = \frac{1}{\rho g}\left\{P_S \pm \left|P_I + P_D + P_R - S_3(x, y, 0)\right|\right\} \quad (13)$$

where η_{r+} represents the maximum relative wave elevation and η_{r-} represents the minimum relative wave elevation.

3 RESULTS AND DISCUSSIONS

In the paper, the trimaran model used by Btizzolara et al (2003) is adopted for the numerical calculation. The definitions of different arrangements of the side hull are shown in Figure 1 and Table 1. For simplicity, only two different arrangements are shown for reference, i.e. PL1PT1 and PL3PT3. In the present study, the trimaran ship is assumed to sail at $Fn=0.3$ in regular head waves with 1m height and the wave frequency with large heave motions, i.e. ω=6.7 rad/sec, is selected to calculate the corresponding unsteady wave.

The wave form, either for steady flow or unsteady motion, and the relative wave elevation along the main hull and side hull are shown in the results for discussion. Figures 2-4 show the wave form along the main hull and side hull for PL1PT1 arrangement, respectively. In Figure 2, the steady wave form along the main hull is small except the bow and stern waves. The unsteady wave amplitude along the main hull is also shown and the peak value appears around the x/L=0.15. In order to consider the critical condition, we combine the positive and negative amplitudes of the unsteady waves with the steady wave to calculate the relative wave elevation along the ship hulls. The results in figure 2 show that the critical value occurs at bow which is more pronounced due to the steady wave and the water shipping on deck may occur. The figures 3 and 4 are the results along the weather beam and leeward of the side hull, respectively. The steady wave forms for both sides of the side hull are similar; however the unsteady waves are significantly different which is due to the different interaction effect from the main hull. Consequently the relative wave elevations for weather beam and leeward of the side hull are different. It is interesting to find that the relative wave elevation is more serious in the leeward which is near the main hull. Therefore it may cause serious impact on the connected deck bottom if the clearance between the water surface and deck bottom is not large enough. The results for another side hull arrangement with Larger ST and CL, i.e. PL3PT3, are shown in figures 5-7. The wave forms and relative wave elevations along the main hull are similar to those for PL1PT1. The fact indicates that the main hull dominates the wave formation. However the effects on the wave form of the side hull are different from those for PL1PT1. In figure 6, we can see the larger relative wave elevation occurs at stern in the weather beam of the side hull and the water shipping may occur if the freeboard of the side hull is not enough. In the leeward, the maximum relative elevation occurs at bow and again the wave impact on connected deck bottom may occur.

4 CONCLUDING REMARKS

The steady and unsteady waves for the trimaran ship traveling at constant speed in waves has been calculated to derive the relative wave elevation with respect to the main hull and side hull. Based on the present analysis for two different side hull arrangements, the following conclusions are drawn :

(1) The steady flow wave can be neglected while the ship speed is low or the incoming wave is large.

(2) The resultant wave form along the main hull is similar with different side hull arrangement; however it is significantly different for the side hull especially in the leeward side.

(3) The large relative wave elevation usually occurs at bow either for main hull or leeward of the side hull, which may cause serious water shipping on deck or impact on the connected deck bottom.

In a word, the different arrangement of clearance and stagger for the side hull of the trimaran affects not only motions and wave loads but also the relative wave elevation. Therefore the suitable selection for the side hull arrangement must be carefully studied overall..

5 ACKNOWLEDGEMENT

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Table 1. The hull configurations.

Configuration	PL0	PL1	PL2	PL3
*Stagger (%)(ST)	0	20	27	40
Configuration	PT0	PT1	PT2	PT3
*Clearance (%)(CL)	9.9	11.1	13.4	15.7

**Stagger: Longitudinal distance between the side hulls transom and main hull transom, in percentage of reference main hull length.*

**Clearance: Lateral distance between the symmetry plane of side hulls and the main hull symmetry plane, in percentage of reference main hull length.*

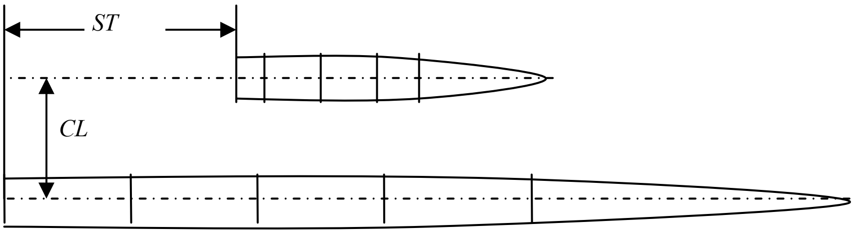

Fig. 1. Diffeent arrangements of the side hull with respect to the main hull.

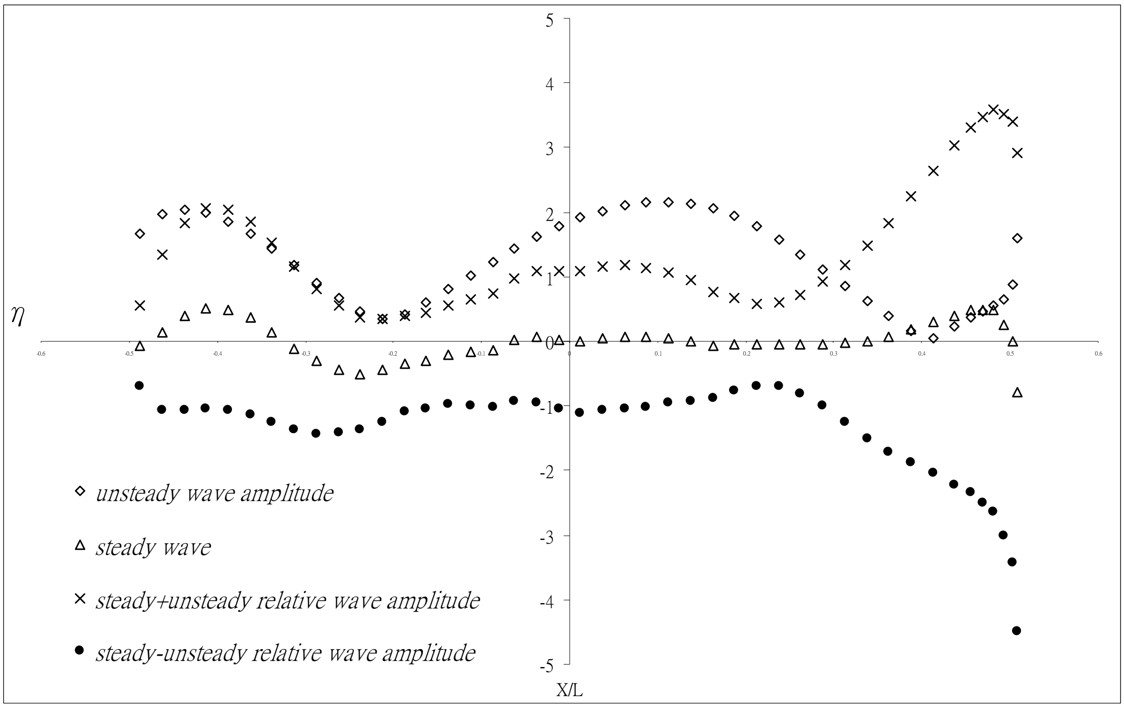

Figure 2. The wave forms and relative wave elevation along the main hull with PL1PT1 in head sea

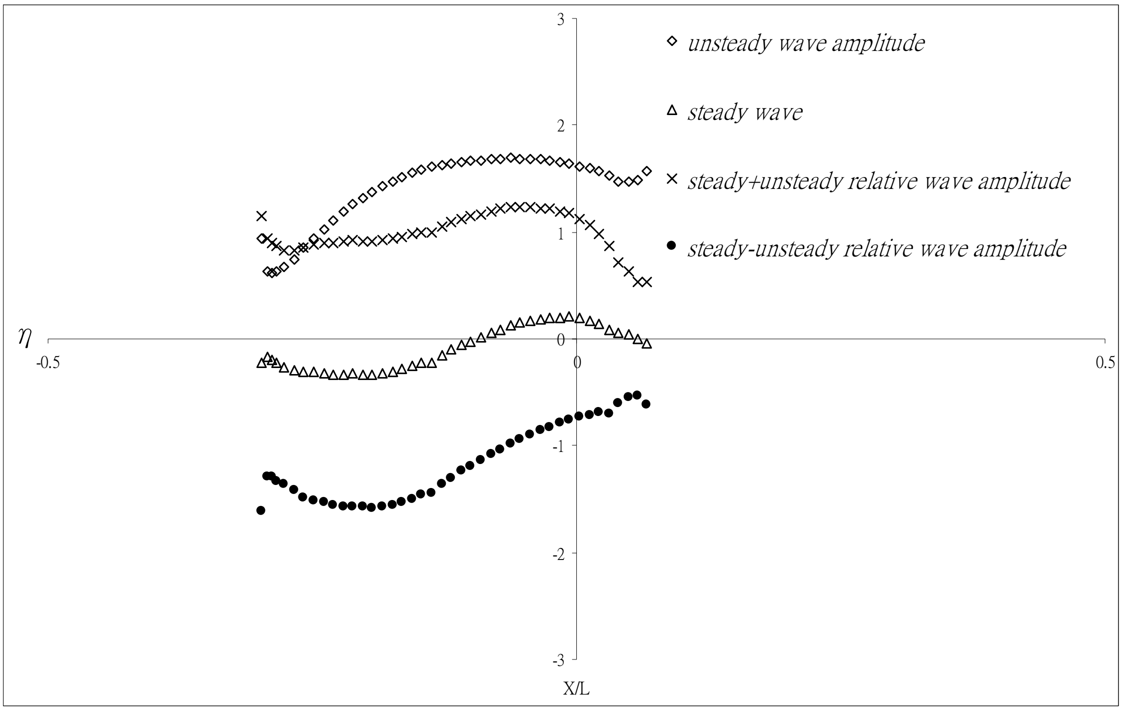

Figure 3. The wave forms and relative wave elevation along the weather beam of the side hull with PL1PT1 in head sea

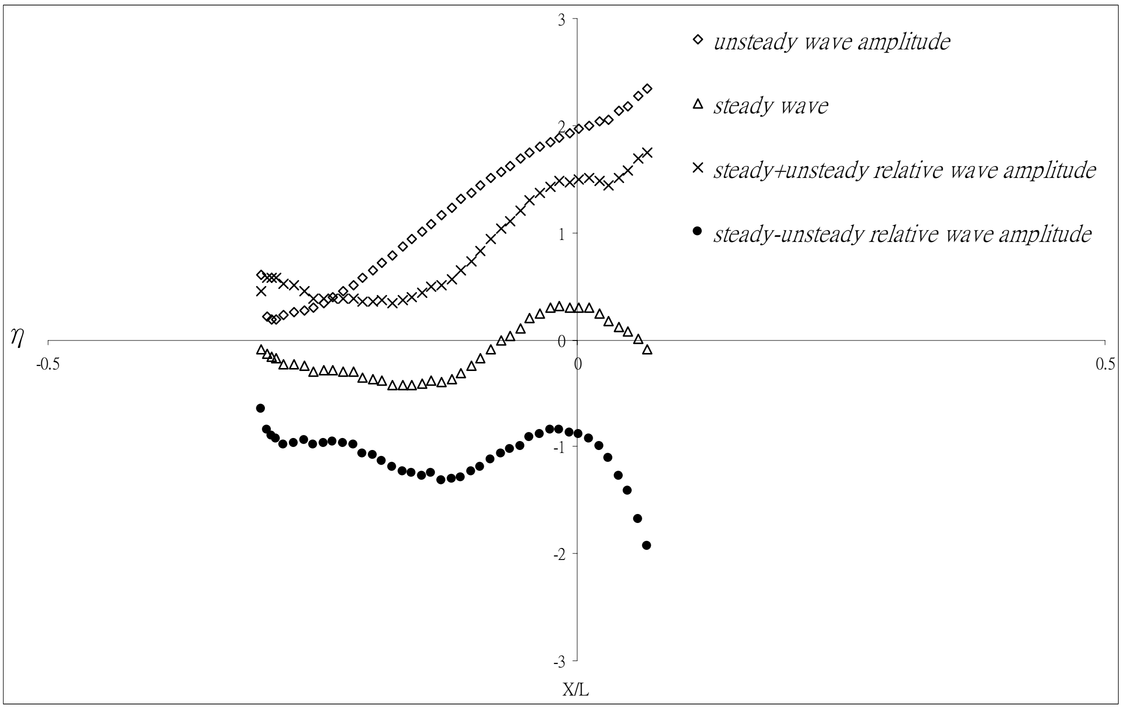

Figure 4. The wave forms and relative wave elevation along the leeward of the side hull with PL1PT1 in head sea

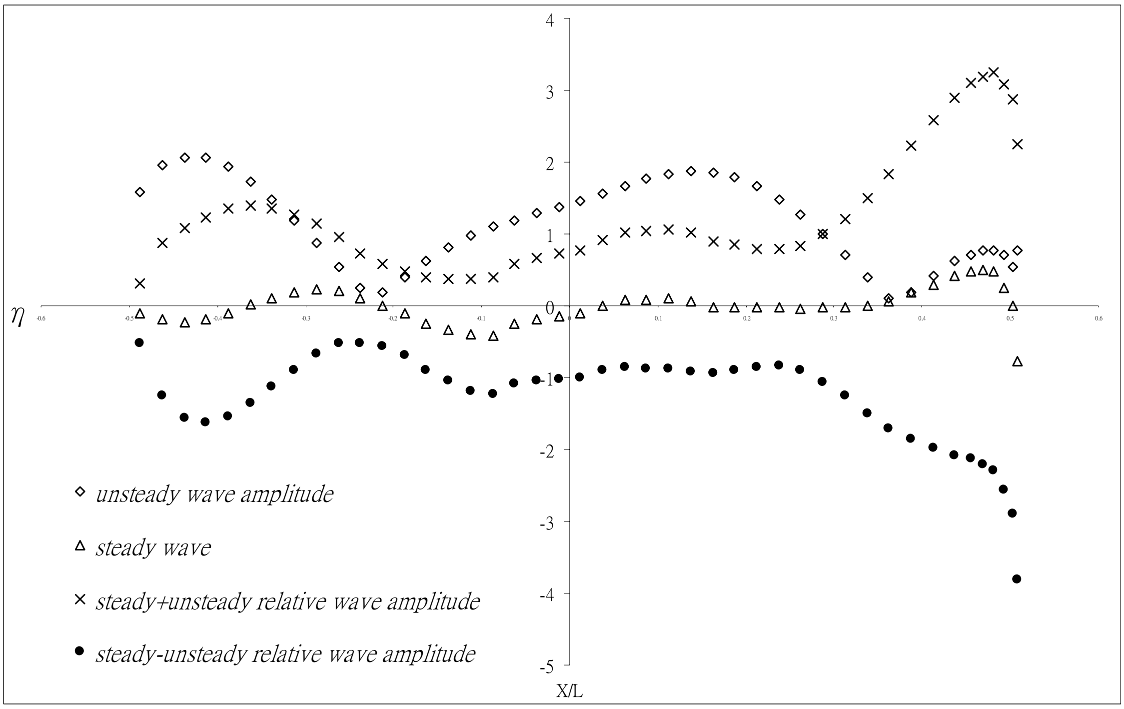

Figure 5. The wave forms and relative wave elevation along the main hull with PL3PT3 in head sea

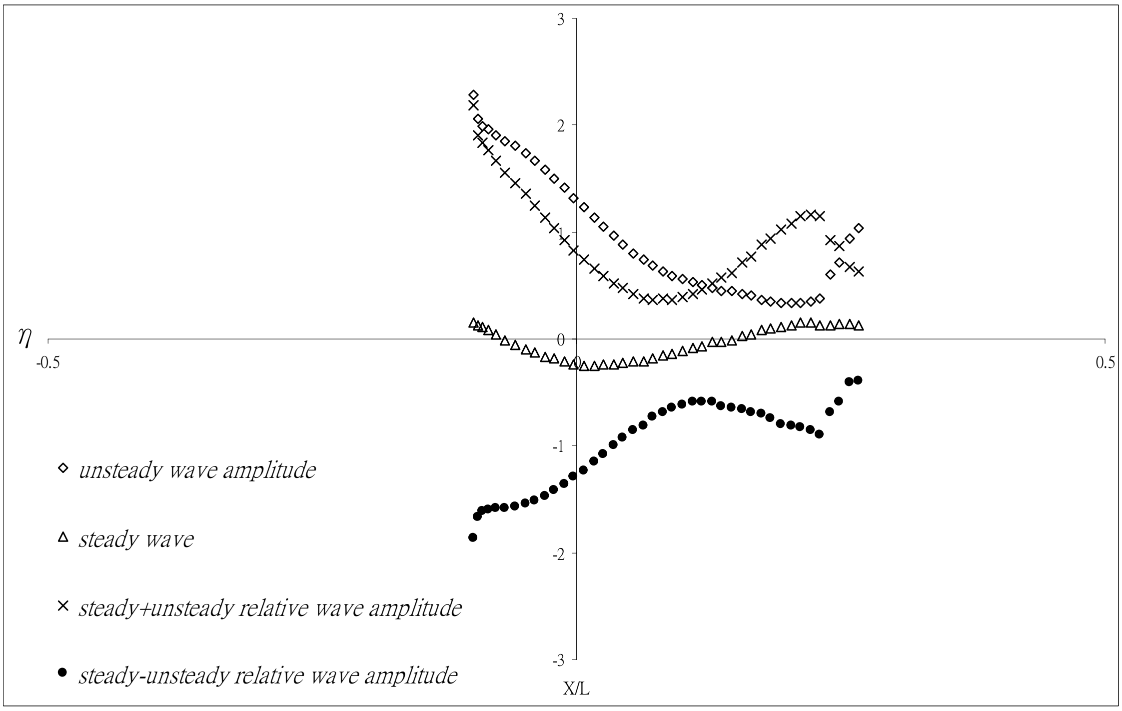

Figure 6. The wave forms and relative wave elevation along the weather beam of the side hull with PL3PT3 in head sea

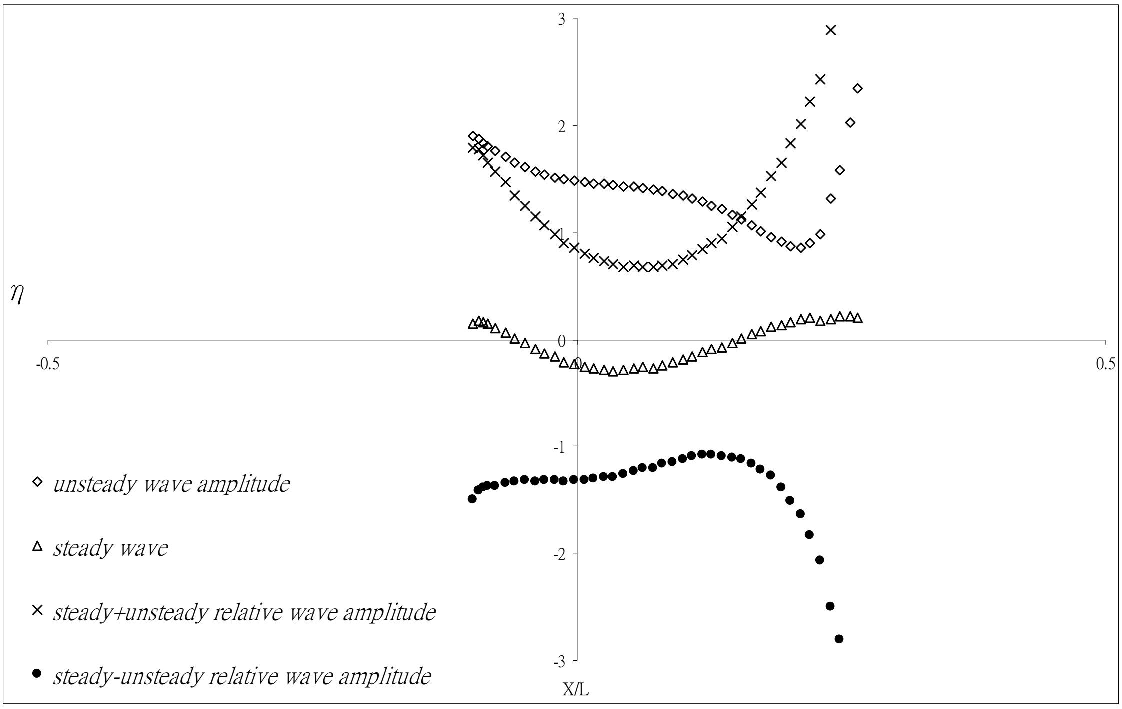

Figure 7. The wave forms and relative wave elevation along the leeward of the side hull with PL3PT3 in head sea